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Chapter 10

The Framework of Complementarities

10.1 Introduction

10.1.1

The world does not present itself to us neatly divided into systems, subsystems, environments, and so on. These are divisions we make ourselves for various purposes. It is evident that different observer-communities find it convenient to divide the world in different ways, and they will be interested in different systems at different times—for example, now a cell, with the rest of the world its environment, and later the postal system, or the economic system, or the atmospheric system. The established scientific disciplines have, of course, developed different preferred ways of dividing the world into environment and system, in line with their different purposes, and have also developed different methodologies and terminologies consistent with their motivations.

Furthermore, throughout this book we have encountered again and again the fact that an observer-community may take alternative views of a system that, at first glance, appear exclusive, but that nevertheless are interdependent and mutually defining. Such was the case with autopoiesis and allopoiesis, and with causal and symbolic explanations, two instances that have been extensively discussed. It was evident through these discussions that keeping the interdependence of these views steadily in mind was a key to a more balanced understanding of natural systems—particularly in the case of autonomy. It is time to recast this issue of interdependence and complementarity of views in a more explicit form.

In this chapter, we present a *conceptual and formal* framework within which a number of various preferred views on systems can be unified. Of particular interest to us here are the differences stemming from the study of natural systems (particularly biological and social systems) and man-made systems (such as engineering and computer systems). Contemporary systems theory has developed extensively through experience in the latter fields, but the insights derived from natural systems have remained by and large much less formally developed. In this book, I hold that the notions of cooperative interaction, self-organization, and autonomy—in brief, holistic notions—are basic to the study of natural systems. In the present framework these notions are not only made more explicit and applicable, but are also presented as *complements* to the more traditional notions of system theory, such as control and input–output behavioral description.

10.1.2

The next section discusses in general terms the role distinction plays in the creation and recognition of systems. The following three sections discuss certain dual perspectives on systems in some detail, including the autonomy/control, state-variable/input–output, holism/reductionism, and net/tree dualities. The fifth section develops the suggestion that such alternatives are complementary rather than antagonistic, into a suggestion that their interrelationship can often be expressed precisely as an adjoint functor relationship, in the sense of categorical algebra. The final section discusses the holism/reductionism relationship in some detail, in relation to the philosophy of science.

Future chapters will build on this foundation, to discuss the notion of autonomy in terms of a (mathematical) theory of self-reference or indefinite recursion and its applications.

10.2 Distinction and Indication

10.2.1

A *distinction* splits the world into two parts, "that" and "this," or "environment" and "system," or "us" and "them," etc. One of the most fundamental of all human activities is the making of distinctions. Certainly, it is the most fundamental act of system theory, the very act of defining the system presently of interest, of distinguishing it from its environment.

Distinctions coexist with purposes. A particularly basic case is autonomy—a system defining its own boundaries and attempting to maintain them; this seems to correspond to what we think of as individuality. It can be seen in individuals (ego or identity maintenance) and in social units (clubs, subcultures, nations). In such cases, there is not only a distinction, but an *indication*, that is, a marking of one of the two distinguished states as being primary ("this," "I," "us," etc.); indeed, it is the very purpose of the distinction to create this indication (Spencer-Brown 1969; Varela, 1975a).

A less basic kind of distinction is one made by a distinctor for some purpose of his own. This is what we generally see explicitly in science, for example, when a discipline "defines its field of interests," or a scientist defines a system that he will study.

In either case, the establishment of system boundaries is inescapably associated with what I shall call a *cognitive point of view*, that is, a particular set of presuppositions and attitudes, a perspective, or a frame in the sense of Bateson (1959) or Goffman (1974); in particular, it is associated with some notion of value, or interest. It is also linked up with the cognitive capacities (sensory capabilities, knowledge background) of the distinctor. Conversely, the distinctions made reveal the cognitive capabilities of the distinctor. It is in this way that biological and social structures exhibit their coherence.

10.2.2

The importance for system theory of cognitive coherence (or the cognitive point of view, or cognitive capability) is a theme that runs throughout this book. Because of the focus on system theory, we shall feel free to invoke the idea of an *observer*, or, observer-community: one or more persons who embody the cognitive point of view that created the system in question, and from whose perspective it is subsequently described.

A simple but fundamental property of the situation involving a system and an observer is that he may choose to focus his attention either on the internal constitution of the system, or else on its environment, taking the system's properties as given. That is, an observer can make a distinction into an indication through the imposition of his value. If the observer chooses to pay attention to the environment, he treats the system as a simple entity with given properties and seeks the regularities of its interaction with the environment, that is, the constraints on the behavior of the system imposed by its environment.[1] This leads naturally to the problem of controlling the behavior of the system, as considered in (engineering) control theory. On the other hand, the observer may choose to focus on the internal structure of the system, viewing the environment as background—for example, as a source of perturbations upon the system's autonomous behavior. From this viewpoint, the properties of the system emerge from the interactions of its components.

Biology has iterated this process of indication, creating a hierarchy of levels of biological study. The cell biologist emphasizes the cell's autonomy, and views the organism of which it is part as little more than a source of perturbations for which the cell compensates. But the physiol-

[1] Calling S "the system" rather than "the environment" already indicates a preference for marking S; that is, the language incorporates the preference. But we may speak of "marking the environment" to suggest that there are in fact two distinct possibilities.

ogist views the cell as an element in a network of interdependences constituting the individual organism: This corresponds to a wider view of environment, namely the ecology in which the individual participates. A population biologist makes his distinctions at a still higher level, and largely ignores the cell. A similar hierarchy of levels can be found in the social sciences. It seems to be a general reflection of the richness of natural systems that indication can be iterated to produce a hierarchy of levels.

At a given level of the hierarchy, a particular system can be seen as an *outside* to systems below it, and as an *inside* to systems above it; thus, the status (i.e., the mark of distinction) of a given system changes as one passes through its level in either the upward or the downward direction. The choice of considering the level above or below corresponds to a choice of treating the given system as autonomous or controlled (constrained). Figure 10-1 illustrates a variety of configurations of systems, subsystems, and marks, and Figure 10-2 illustrates the hierarchy of levels.

10.3 Recursion and Behavior

10.3.1

In system theory, the autonomy/control distinction appears more specifically as a recursion/behavior distinction. The behavioral view reduces a system to its input-output performance or behavior, and reduces the environment to inputs to the system. The effect of outputs on environment is not taken into account in this model of the system. The recursive view of a system, as expressed in the closure thesis, emphasizes the mutual interconnectedness of its components (von Foerster, 1974; Varela, 1975a; Varela and Goguen, 1978). That is, the behavioral view arises when emphasis is placed on the environment, and the recursive view arises when emphasis is placed on the system's internal structure.

If we stress the autonomy of a system S_i (see Figure 10-1), then the environmental influences become perturbations (rather than inputs) which are compensated for through the underlying recursive interdependence of the system's components (the S_{i-1}'s is the figures). Each such component, however, is treated behaviorally, in terms of some input-output description.

The recursive viewpoint is more sophisticated than the behavioral, since it involves the simultaneous consideration of three different levels, whereas the behavioral strictly speaking involves only two. This is because the behavioral model, in taking the environment's view of the system, does not involve making any new distinctions. But expressing interest in how the system achieves its behavior through the interdependent action of its parts adds a new distinction, between the system and its parts.

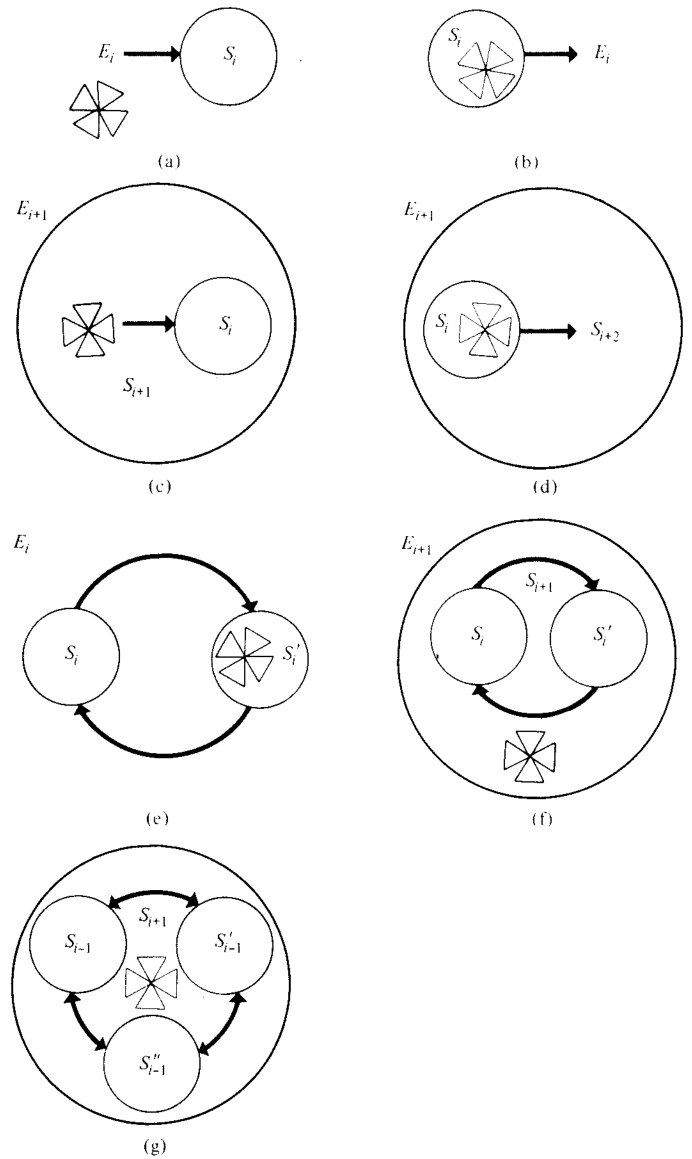


Figure 10-1
Various configurations of systems, subsystems, and marks; Each configuration represents a cognitive viewpoint, and the mark indicates its center. The arrows indicate the interactions.

From Goguen and Varela (1978a).

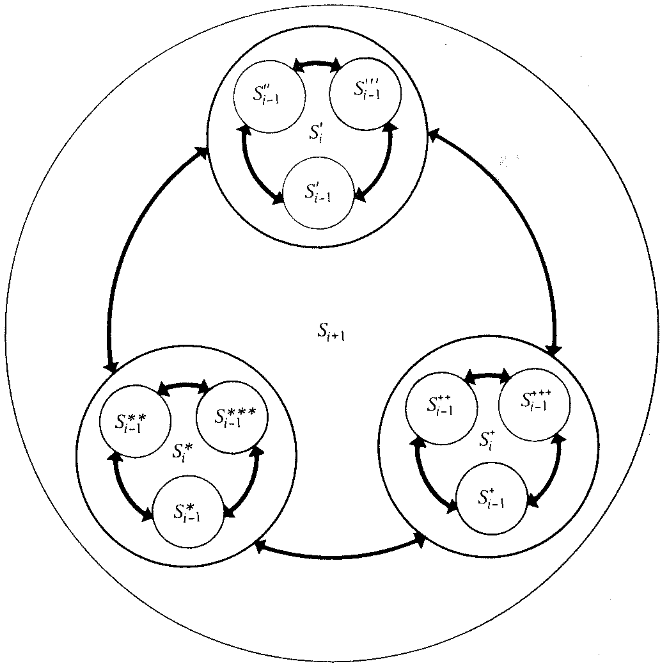


Figure 10-2
Diagrammatic evocation of a hierarchy of system levels. See text for further discussion.

From Goguen and Varela (1978a).

10.3.2

The following may help to make this seem less abstract. The most traditional way to express the interdependence of variables in a system is by differential equations (cf. Section 7.2.4). An autonomous system can be formally represented by equations of the form

$$\dot{x}_i = F_i(x, t) \quad \text{for} \quad 1 \leq i \leq n, \tag{10.1}$$

where $x = (x_1, \ldots, x_n)$ is the state vector of the system. The autonomous behavior of the system is described by a solution vector $x(t)$ that satisfies (10.1). This involves treating everything as happening on the same level, and all variables as being observable; in effect, the environment is treated as part of the system (or ignored).

However, the effect of the environment on the system can be represented by a vector $e = (e_1, \ldots, e_k)$ of parameters, giving

$$\dot{x}_i = F_i(x, e, t) \quad \text{for} \quad 1 \leq i \leq n, \tag{10.2}$$

which explicitly takes account of two levels. Solutions to the system (10.2) are now also parametrized by e, that is, they are of the form $x(e, t)$.

The situation of (10.2) can be elaborated in two directions. In control theory, it is usual to assume that the internal variables x of the system are either unobservable or of no direct interest, and that we have instead direct access to (or interest in) an output vector y of variables that are functionally dependent on x. The variables e are usually taken to be under the control of the observer, and the question is posed, how to use those variables to obtain certain desired values of y. The equations are thus of the form

$$\dot{x}_i = F_i(x, e, t), \qquad y = H(x). \tag{10.3}$$

Strictly speaking, the equations span three levels, and can be used, for example, to infer information about the system's internal state, but the emphasis ("mark") is on the environment, which is identified with the observer. Behavior appears as an input-output function $y(e, t)$, the observable results of applying the inputs (also called "controls") e to the system.

An alternative elaboration of the situation of (10.2) views the vector e as not necessarily or particularly under the control of an observer, but rather as a source of perturbations upon (10.2). For example, the components e_j of e may be some coefficients, which are regarded as constants in the original equation (10.1). A natural question to pose is the stability of the system under such perturbations, that is, the relation of (10.1) to a perturbed system

$$\dot{x}_i = F_i(x, e, t) + \delta F_i(x, e, t) \tag{10.4}$$

in which δ (in a fairly intuitive notation) represents a "small change." It is known, for example, that changes in structural constants can cause the system to undergo a "catastrophic" change [in the sense of Thom (1972)] into a new configuration.

The system (10.2) has in it nothing that intrinsically prefers the approach of either (10.3) or (10.4). This choice depends on the interest of the analyst.

Note that recursion plays a role in all these formulations, but is more obscure in the control-theory interpretation. On the other hand, the behavioral information, though still available, is more obscure in the stability interpretation (10.4). We are not, of course, claiming that either of these approaches is inherently better.

10.3.3

Historically speaking, some of the many possible approaches to systems have been much more developed than others. The most highly developed parts, in fact, center on the notions of control, input-output behavior, and state transition. This is presumably because of the interest in applying these approaches in engineering.

The notion of autonomy, however, is particularly important for natural systems (biological and social systems), and the lack of a well-developed theory of autonomous systems is a serious difficulty. An engineer designing an artifact will choose the inputs of interest to him for this application with some assurance that the choice will be adequate. But a biologist studying a cell is forced to acknowledge the autonomy of the cell; if the biologist's preferences for input and output variables do not match the cell's internal organization, his cognitive-domain theory will be useless. Furthermore, the hierarchy of levels seems to particularly assert its importance for natural systems, so that it is generally necessary to take account of at least three levels. Even when the lowest level is very well understood, the role it plays at the next higher level, where it is interconnected with other systems, can be quite obscure. An enzyme biochemist may be able to describe a particular metabolic loop very effectively by a transfer function, but be quite unable to specify how it fits into the overall metabolic process of the cell as a coherent autopoietic whole.

This situation of being unable to understand how elements, even quite well-understood elements, coordinate or somehow function effectively together at the next higher level, is quite common in the study of natural systems, and is another source of our motivation for a better-developed theory of autonomous systems.

10.3.4

Some fragments of a theory emphasizing the autonomy of systems do exist, but are far less developed than the computer-gestalt, behavioristic approach. In fact, the dominance of control views in contemporary systems theory makes it closer to a theory of system components than to one of systems as unities (totalities). Let's mention briefly some existing approaches to representing, in formal terms, some of the characteristics of autonomy.

First and foremost, the idea of stability derived from classical mechanics has been extensively studied and used. As we said before (Section 7.2.4), a set of interdependent differential equations can be used to represent the autonomous properties of a whole system. Rosen (1972), Iberall (1973), and Lange (1965) have applied this perspective to natural systems with various degrees of emphasis on autonomous behavior. More specific examples can be found in population biology (May, 1971), in molecular biology (Eigen and Schuster, 1978; Goodwin, 1976; Rössler, 1978; Bernard-Weil, 1976), and more recently in neurobiology (Katchal-[illegible] Rowland and Blumenthal, 1974; Freeman, 1975). Some thought has

been given to cooperative interactions in this area of hierarchical multilevel systems. The idea of hierarchy is often presented from the point of view of the interdependence of different levels of system descriptions (Pattee, 1972; Whyte et al., 1968; Mesarovic et al., 1972). Particular instances of hierarchical structure, including multilevel cooperation, can be found in Beer (1972), Kohout and Gaines (1976), and Baumgartner et al. (1976). Goguen (1971, 1972) presents a general theory of hierarchical systems of interdependent processes. Its basic ideas are interconnection, behavior, and level, and its theoretical framework is categorical algebra. A last area in which the idea of a whole system is somewhat explicit is that of self-organizing systems. Work in this area, based on an information-theoretic approach, includes von Foerster (1966) and Atlan (1972, 1978).

We do not intend to unite *all* these various threads of research together in a single framework. Rather, we emphasize the ways in which pairs of seemingly different points of view, such as autonomy/control, are complementary, in the sense of contributing to a better understanding of natural systems. But the idea of complementarity, fundamental though it seems, is still vague. The following develops an *explicit* definition.

10.4 Nets and Trees

10.4.1

If we retain interest only in the connectivity of a system, it is possible to *represent the recursion/behavior duality by a network/tree duality*. Intuitively, the nodes in these nets or trees represent the elements or components of a system, while their links represent interactions or interconnections. The reciprocal connectivity of a net suggests the coordination of a system's elements; a tree structure suggests the sequential subordination of a system's parts, each part having its own well-defined input-output behavior description. To be sure, in retaining only the basic connectivity of a system's organization, much is discarded in the net/tree representation. We intend to use this convenient general representation to study complementarity.

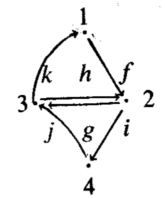
Now to the definition of nets and trees. Let there be a set $\{v_1, \ldots, v_n\}$ of nodes (components or parts), which are to be interconnected by a set $E = \{e_i, \ldots, e_p\}$ of edges (relations or processes).

Definition 10.1

A network is a directed graph G, that is, a quadruple $G = \langle |G|, E, \partial_0, \partial_1 \rangle$, *where* $|G| = \{v_1, \ldots, v_n\}$, *and* $\partial_i\colon E \to |G|$ *are the source* $(i = 0)$ *and target* $(i = 1)$ *functions, from the edges to the nodes of G. If* $e \in E$, $\partial_0 e = v$, *and* $\partial_1 e = v'$, *then we write* $e\colon v \to v'$.

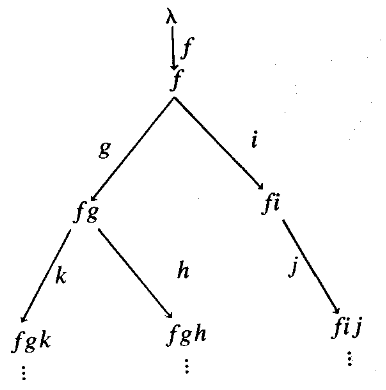
Definition 10.2
A path from v to v' in a graph G is a finite sequence $p = e_0 \ldots e_n$ of edges that are adjacent, that is, satisfy $\partial_1 e_i = \partial_0 e_{i+1}$ for $1 \le i \le n$, with $\partial_0 e_0 = v$ and $\partial_1 e_n = v'$. If $\partial_0 p = v$ and $\partial_1 p = v'$, then we write $p\colon v \to v'$.

Example 10.3
Consider the graph G:



(10.5)

The nodes 1, 2, 3, 4 might represent four physical locations, each with a radio. Because of differences in transmitter and receiver strength, available frequencies, and terrain, communication is possible only along the channels indicated by arrows in (10.5). For example, there is a mountain between 1 and 4. Let us assume that node 1 is of particular interest—say it is our base. Then we are interested in the patterns of transmission which are possible starting from our base. For example, to reach node 3, if channel g is out, we can send a message via fij; to verify its correctness, it could be sent back to node 1, via the path $fijk$; node 3 might also generate a reply, which would require a message to be sent to node 2, giving a path $ijkf$; and so on. Thus, we are interested in the set of all paths in G with source 1. This collection itself has a branching structure, because starting with a given path, it can sometimes be developed further by choosing alternative edges to get alternative paths. The collection of all choices can be represented by the following tree:



(10.6)

In some sense the tree (10.6) "unravels" or "unfolds" the graph (10.5) from node 1. To make this more precise we need to define tree, pointed graphs, and the idea of structure-preserving mappings of graphs called graph homomorphism. First we give the general construction.

Definition 10.4
A pointed graph G is a 5-tuple $\langle |G|, E, \partial_0, \partial_1, a\rangle$ such that $\langle |G|, e, \partial_0, \partial_1\rangle$ is a graph and $a \in |G|$ is a vertex.

A pointed graph is reachable if for each vertex $v \in |G|$ there is a path $a \to v$ in g.

A graph G is loop-free if for all $v, v' \in |G|$ there is at most one path $v \to v'$.

A tree is a reachable loop-free pointed graph.

Definition 10.5
Let G be a pointed graph $\langle |G|, E, \partial_0, \partial_1, a\rangle$. Then the unfoldment $U_a(G)$ of G from a is the graph in which: $|U_a(G)|$ is all the paths $p\colon a \to v$ for $v \in |G|$; the edges of $U_a(G)$ are the pairs $\langle p, pe\rangle$ such that $p, pe \in |U_a(G)|$, and $e \in E$; $\partial_0\langle p, pe\rangle = p$, and $\partial_1\langle p, pe\rangle = pe$. The null path $a \to a$ is written $1_a\colon a \to a$, and is taken to be the point for $U_a(G)$.

Proposition 10.6
Let G be a pointed graph $\langle |G|, E, \partial_0, \partial_1, a\rangle$, and $U_a(G)$ the unfoldment of G from a. Then $U_a(G)$ is a tree.

PROOF: We must show that $U_a(G)$ is a pointed, reachable, and loop-free graph. By the definition of $U_a(G)$, we know that it is a pointed graph, with point $1_a\colon a \to a$.

We now show that $U_a(G)$ is reachable. Consider a node $p\colon a \to v$ of $U_a(G)$, say $p = e_0 \ldots e_n \ne 1_a$, with $e_i \in E$. Then we can show that

$$q = \langle 1_a, e_0\rangle\langle e_0, e_0e_1\rangle\langle e_0e_1, e_0e_1e_2\rangle, \ldots, \langle e_0 \ldots e_{n-1}, e_0 \ldots e_n\rangle$$

is a path from 1_a to p in $U_a(G)$. Clearly its source is 1_a, since

$$\partial_0 q = \partial_0\langle 1_a, e_0\rangle = 1_a,$$

and its target is p, since

$$\partial_1 q = \partial_1\langle e_0 \ldots e_{n-1}, e_0 \ldots e_n\rangle = e_0 \ldots e_n = p.$$

Moreover, q is a path, since its edges are adjacent, that is,

$$\begin{aligned}\partial_1\langle e_0 \ldots e_k, e_0 \ldots e_{k+1}\rangle &= \partial_0\langle e_0 \ldots e_{k+1}, e_0 \ldots e_{k+2}\rangle \\ &= e_0 \ldots e_{k+1} \quad \text{for} \quad 0 \le k \le n-1.\end{aligned}$$

So there is a path from 1_a to every node in $U_a(G)$.

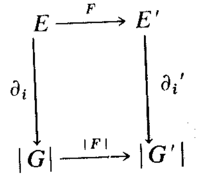
Last we show that $U_a(G)$ is loop-free, that is, for every pair of nodes p, p' in $U_a(G)$ there is at most one path $p' \to p$ in $U_a(G)$ with target p. Consider again $p = e_0 \ldots e_n$, and let's show first that there is exactly one edge with target p, e.g., exactly one path of length one. Any edge with target p is of the form $\langle r, re_n \rangle$, with $re_n = p$. Thus r must equal $e_0 \ldots e_{n-1}$, and the unique edge is $\langle e_0 \ldots e_{n-1}, p \rangle$. This says that if $p' \neq p$, a path $p' \to p$ must end with edge $\langle e_0 \ldots e_{n-1}, p \rangle$. Let now $p_k = e_0 \ldots e_k$. Then a path $p' \to p$ must be a composite of a path $p' \to p_{n-1}$ with edge $\langle p_{n-1}, p \rangle$. But we may reason in a similar way for the node p_{n-1}, and the path $p' \to p_{n-1}$, p_{n-2}, and $p' \to p_{n-2}$, and so on. Eventually we must find that $p' = p_k$ for some k, and the unique path $p' \to p$ is of the form

$$\langle p_k, p_{k+1} \rangle \langle p_{k+1}, p_{k+2} \rangle \ldots \langle p_{n-1}, p \rangle .$$

If $p = p'$, the unique path $p \to p'$ is the null path at p. Thus $U_a(G)$ is loop-free, and the proof is complete. □

Definition 10.7

Let $G = \langle |G|, E, \partial_0, \partial_1 \rangle$ and $G' = \langle |G'|, E', \partial_0', \partial_1' \rangle$ be graphs. Then a graph morphism is a pair $\langle |F|, F \rangle$ of functions $|F|: |G| \to |G'|$ and $F: E \to E'$, such that the source and target relationships are preserved, that is, such that $\partial_0'(F(e)) = |F|\partial_0(e)$, and $\partial_1'(F(e)) = |F|\partial_1(e)$; i.e., such that the diagram



commutes for $i = 0$ and $i = 1$. We abbreviate $\langle |F|, F \rangle$ to just F. A morphism of pointed graphs, from G to G', is a graph morphism F such that $|F|a = a'$, where a, a' are the selected vertices of G, G' respectively.

10.4.2

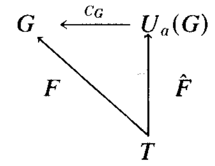
The relation between a graph G and its unfoldment is, from our perspective, very interesting. Given a node a in G, then $U_a(G)$ is a loop-free version of G. We could say $U_a(G)$ expresses G as a (possibly infinite) chain of subordinated choices, starting from the selected node. The unfoldment of G optimally "covers" G in a sense that is made precise through the "universal property" of $U_a(G)$, that any graph morphism F: ... factored through a "covering morphism" C_G: $U_a(G) \to$

G, defined as follows: for p a node of $U_a(G)$, let $|C_G|p = \partial_1 p$; and for $\langle p, pe \rangle$ an edge of $U_a(G)$, let $C_G\langle p, pe \rangle = e$.

We now show that C_G is a graph morphism. For $\langle p, pe \rangle$ an edge of $U_a(G)$, then $\partial_0 C_G\langle p, pe \rangle = \partial_0 e$, $|C_G|\partial_0\langle p, pe \rangle = |C_G|p = \partial_1 p$, and $\partial_0 e = \partial_1 p$ because pe is a path. Also, $\partial_1 C_G\langle p, pe \rangle = \partial_1 e$, and $|C_G|\partial_1\langle p, pe \rangle = |C_G|pe = \partial_1 pe = \partial_1 e$. We now show that any other morphism from a tree can be factored through C_G.

Theorem 10.8

Let G be a pointed graph, let T be a tree, and let F: $T \to G$ be a pointed graph morphism. Then there is a unique pointed graph morphism $\hat{F}$: $T \to U_a(G)$ such that



commutes.

Sketch of proof: Let $v \in |T|$. Then there is a unique path p: $r \to v$ in T, where r is its root, and F_p: $Fr = a \to Fv$ is a path in G. Now we define $|\hat{F}|v = F_p$, and for e: $v \to v'$, an edge in T, we define $\hat{F}e = \langle F_p, (F_p)Fe \rangle$, an edge in $U_a(G)$. Then

$$|C_G|\,|F|v = |C_G|F_p = F\partial_1 p = |F|v,$$

and

$$C_G\hat{F}e = C_G\langle F_p, (F_p)Fe \rangle = Fe.$$

Thus

$$C\hat{F} = F.$$

We now verify that $\hat{F}$ is a pointed graph morphism:

$$\partial_0\hat{F}e = \partial_0\langle F_p, (F_p)Fe \rangle = F_p,$$
$$|\hat{F}|\partial_0 e = |\hat{F}|v = F_p, \quad \text{and} \quad \hat{F}r = F1_r = 1_a;$$
$$\partial_1\hat{F}e = \partial_1\langle F_p, (F_p)Fe \rangle = (F_p)Fe = F(pe) = |\hat{F}|\partial_1 e.$$

Thus $\hat{F}$ satisfies the required conditions. [For a detailed proof and the uniqueness argument, see Goguen (1974, Theorem 7).] □

10.4.3

This theorem brings into focus the basic intuition that there is a mutual interdependence between a system's elements (as a graph) and the sequential subordination of their interconnections (as a tree). To express this more clearly, let $\mathcal{G}r^*$ be the class of pointed graphs, and let $\mathcal{T}$ be the class of trees. Also, if G and G' are pointed graphs, let $\mathcal{G}r^*(G, G')$ denote the set of all pointed-graph morphisms from G to G'.

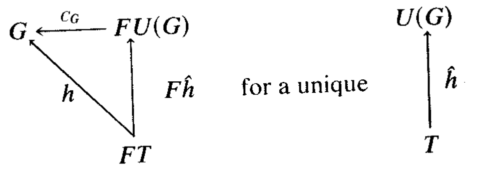
Since every tree is by definition a pointed graph, we have a mapping

$$\mathcal{T} \xrightarrow{F} \mathcal{G}r^*$$

which simply views trees as pointed graphs. We also have a mapping

$$\mathcal{G}r^* \xrightarrow{U} \mathcal{T}$$

which assigns to every pointed graph the tree $U(G)$ that covers it. These two mappings F, U are tightly interlocked. For any $h \in \mathrm{Mor}(G, FT)$, the set of morphisms from G to FT, we have



by Theorem 8. This says that there is a bijection

$$\varphi: \mathcal{G}r^*(FT, G) \to \mathcal{G}r^*(T, UG)$$

defined by $\varphi(h) = \hat{h}$. We shall call (F, U, φ) a *complementarity* between $\mathcal{G}r^*$ and $\mathcal{T}$. This notion of net/tree complementarity effectively relates two levels of description of systems, in such a way that each necessitates the other. It is convenient at this point to see that similar notions of complementarity apply to other situations, and so we now turn to the general notion.

10.5 Complementarity and Adjointness

10.5.1

The net/tree complementarity is a particularly clear instance of the interdependence of apparent dualities. This section develops this idea in the general setting of category theory, which is becoming increasingly useful in systems theory (Goguen, 1973; Arbib and Manes, 1974). Readers unfamiliar with this terminology may find a leisurely introduction in ADJ (1973, 1976) or Arbib and Manes (1974); we attempt to stay at a fairly intuitive level, although some technicalities are inevitable.

The intuitive idea of a *category* is that it embodies some structure by exhibiting the class of all *objects* having that structure, together with all the structure-preserving mappings or *morphisms* among them. (Somewhat more technically, categories assume there is an associative opera-

tion of composition on those morphisms whose source and target match.) The idea is due to Eilenberg and MacLane (1945).

For example, pointed graphs and pointed-graph morphisms constitute a category. If $\mathcal{C}$ is a category, and A, B are objects in $\mathcal{C}$, we shall let $\mathcal{C}(A, B)$ denote the set of all morphisms in $\mathcal{C}$ from A to B.

Usually, we are interested not only in objects from various categories, but even more in certain constructions performed on the objects of one category to yield objects of another category. For example, unfoldment is a construction performed upon graphs that yields trees. This construction has a kind of consistency, in that it can also be extended to the morphisms; that is, a morphism of pointed graphs induces, in a natural way, a morphism between their unfoldments. This kind of consistency is expressed by saying that the construction is *functorial,* or is a *functor.* (More technically, this has to do with the preservation of the composition of morphisms.)

However, the unfoldment construction is natural in a much stronger sense: the "optimal" covering of a graph is its unfoldment; this is expressed by the universal property of Theorem 8, and the bijection φ of the previous section. The concept of *adjunction* generalizes just this state of affairs to the following situation:

Let $\mathcal{A}$ and $\mathcal{B}$ be categories, let F be a functor from $\mathcal{A}$ to $\mathcal{B}$, and G a functor from $\mathcal{B}$ to $\mathcal{A}$. Then an *adjunction* is, in addition, a natural bijection

$$\varphi: \mathcal{B}(FA, B) \to \mathcal{A}(A, GB).$$

This says that every morphism f: $FA \to B$ determines a unique morphism $\varphi(f)$: $A \to GB$. [The precise sense of the "naturalness" of φ is that of natural transformation, due to Eilenberg and MacLane (1945)—which, however, we shall not define here. The idea of adjunction is due to Kan (1958).]

The discussion at the end of the previous section shows that the net/tree complementarity is an instance of the concept of adjunction. *What we now propose is to explore the view that the precise concept of adjunction is an application of the general (and vague) concept of complementarity.*

10.5.2

Another example of this is Goguen's (1973) adjunction between minimal realization and behavior. Let $\mathcal{A}$ be the category of automata (in some fixed sense that we shall not explain in detail), and let $\mathcal{B}$ be the category of input-output behaviors of such automata (with appropriate morphisms). Then there is a functor from $\mathcal{B}$ to $\mathcal{A}$ that constructs the minimal automation $M(B)$ having the behavior B; and there is a functor Be from $\mathcal{A}$ to $\mathcal{B}$ that constructs the behavior Be(A) of an automaton A. Moreover, there is a natural bijection

$$\varphi: \mathcal{B}(\mathrm{Be}(A), B) \to \mathcal{A}(A, M(B))$$

that expresses the complementarity of the notions of internal state transition (as embodied in automata) and input-output behavior. Goguen (1972) has shown that many other classes of systems exhibit such a complementarity with their input-output behaviors.

Here is still another example. If G is a graph, the collection of all paths (from all sources) in G forms a category whose objects are the nodes of G, and whose morphisms are the paths of G; this category is denoted $\mathcal{P}a(G)$, and called the *path category* of G. $\mathcal{P}a$ is a functor from the category $\mathcal{G}r$ of graphs to the category $\mathcal{C}at$ of (small) categories. There is also a functor F from $\mathcal{C}at$ to $\mathcal{G}r$, which merely forgets the additional structure that categories have over graphs (namely, the possibility of composing morphisms), regarding the objects as nodes, and the morphisms as edges. Again, there is a natural bijection

$$\varphi: \mathcal{C}at(\mathcal{P}a(G), \mathcal{C}) \to \mathcal{G}r(G, F\mathcal{C})$$

expressing the complementarity of graphs and categories. Alternatively, $\mathcal{P}a(G)$ is the free category generated by the graph G, and the adjunction (or the corresponding "universal property") expresses this relationship (Goguen, 1974).

Lawvere (1969), in a particularly fundamental paper, suggests that there is a complementary relationship between the traditional conceptual and formal viewpoints in the foundations of mathematics. This duality also appears as a semantic syntax pair, in that Lawvere (1963) has shown an adjunction between a functor that associates with each algebraic theory its category of semantic models (i.e., its algebras), and a functor that extracts from each category the optimal syntactic theory of its algebraic component of structure.

The general system theory of Goguen (1971, 1972) involves a hierarchy of levels, much as pictured in Figure 10-2, with functors going outward if they regard a component at a lower level as a whole system at the next higher level, and functors going inward if they compute the behavior of the whole system, viewing the result as a single object at the lower level. There is a base level of given "objects" out of which systems can be constructed, and objects at level $i + 1$ are interconnections (that is, systems) of objects at level i. Goguen shows that each pair of an outward and an inward functor is an adjunction. The inward functor is in fact the fundamental categorical construction known as "limit." Goguen also shows that the construction of interconnecting a system of systems (over some common subparts as "terminals") to get a single system, is given by the dual concept of "colimit," which also appears as an adjunction.

This is not the place to give details, but the connection with themes of this book should be evident.

This general point seems particularly clear in the context of systems theory: There is no whole system without an interconnection of its parts; and there is no whole system without an environment. Such pairs are mutually interdependent: each defines the other. What is remarkable about the notion of adjoint functor is that it captures the notion of complementarity in a very precise way, without imposing any particular model for the nature of the objects so related. It is also worth noting that there is a well-developed theory of adjunctions; for example, the composition of two adjoint pairs of functors is another adjoint pair. Of course, not all pairs of descriptive modes are complementary, and similarly, not all pairs of functors are adjoint. The so-called "adjoint functor theorem" provides some general conditions for when a given functor in fact does have an adjoint; and again, this may well find some application in general discussions about system theory. Much more work, including many further examples, will be needed to discover the proper domain of application, and the limits, of the adjointness idea.[2]

10.6 Excursus into Dialectics

10.6.1

In general, when different modes of description appear as opposites, it is more satisfactory to consider them as complementary instead. This is the case, quite rigorously, with the apparent dualities net/tree and recursion/behavior, as we have seen above. On a more intuitive level, there is a similar relationship for the pairs autonomy/control and operational/symbolic discussed in earlier sections. As a matter of fact, we may go one step further to duality and dialectics as a broad philosophical idea. Accordingly, I would like to go into a brief excursus to discuss trinities.

By trinity I mean the consideration of the ways in which pairs (poles, extremes, modes, sides) are related yet remain distinct—the way they are not one, not two (Varela, 1976). The key idea here is that we need to replace the metaphorical idea of "trinity" with some built-in injunction (heuristic, recipe, guidance) that can tell us *how* to go from duality to trinity:

$$* = \text{the it / the process leading to it.}$$

The slash in this *star* ($*$) statement is to be read as: "consider both sides of the /," that is, "consider both the it and the process leading to it."

[2] We have not discussed at all the notion of complementarity in physics, and whether the present framework is applicable. To do so is completely beyond my competence.

Thus the slash here is to be taken as a compact indication of a way of transiting to and from both sides of it.

We can now transcribe the familiar relationship between nets/trees into a star form:

$$* = \text{network / trees constituting the network},$$

because the duality is connected with processes in both directions quite explicitly. The totality (the net) is seen as emerging of resulting from part-by-part approximation of the trees (the process leading to it).

Similarly we may consider a more generally appealing star:

$$* = \text{whole / parts constituting the whole}$$

By a whole, a totality here we mean a simultaneous interactions of parts (components, nodes, subsystems) that satisfies some criteria of distinction. Thus a star of a more operational flavor is

$$* = \text{stability / approximation in time.}$$

Let us formulate a number of other interesting dualities in this complementarity framework, informally called star. To this end, take any situation (domain, process, entity, notion) that is autonomous (total, complete, stable, self-contained), and put it on the left side of the /. Put on the other side the corresponding process (constituents, dynamics). For example:

being/becoming	environment/system
space/time	context/text
reality/recipe	semantics/syntax
simultaneous/sequential	autonomy/control
arithmetic/algebra	symbolic/operational
analog/digital	

In each of these cases the dual elements can be seen as complementary: they *mutually specify each other*. There is, in this sense, no more duality, since they are related.

10.6.2

Notice that this separation of duality is no "synthesis" (in the Hegelian sense), since there is really nothing "new," but just a more direct appraisal of how things are put together and related through our descriptions, not losing track of the fact that every "it" can be seen on a different level as a process.

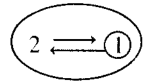
More generally, we may see that this view of complementarity signifies a departure from the classical way of understanding dialectics. In the classical (Hegelian) paradigm, duality is tied to the idea of polarity, a

clash of opposites. Graphically,

$$① \rightleftarrows ② \ .$$

The basic form of these kinds of duality is symmetry: Both poles belong to the same level. The nerve of the logic behind this dialectics is negation; pairs are of the form *A*/not-*A*.

In this presentation, dualities are adequately represented by *imbrication* of levels, where one term of the pair *emerges* from the other. Graphically,



The basic form of these dualities is asymmetry: Both terms extend across levels. The nerve of the logic behind this dialectics is self-reference, that is, pairs of the form: it / process leading to it.

Pairs of opposites are, of necessity, on the same level and stay on the same level for as long as they are taken in opposition and contradiction. Pairs of the star form make a bridge across one level of our description, and they specify each other. When we look at natural systems, nowhere do we actually find opposition except from the values we wish to put on them. The pair predator/prey, say, does not operate as excluding opposites, but both generate a whole unity, an autonomous ecosystemic domain, where there are complementarity, stabilization, and survival values for both. So the effective duality is of the star form: ecosystem/species interaction.

We may generalize this to say that there is an *interpretative rule* for dualities:

For every (Hegelian) pair of the form A/not-A there exists a star where the apparent opposites are components of the right-hand side.

It is, I suspect, only in a nineteenth-century social science that the abstraction of the dialectics of opposites could have been established. This also applies to the observer's properties. We have maintained all along that whatever we describe is a reflection of our actions (perceptions, properties, organization). There is mutual reflection between describer and description. But here again we have been used to taking these terms as opposites: observer/observed, subject/object as Hegelian pairs. From my point of view, these poles are not effectively opposed, but moments of a larger unity that sits on a metalevel with respect to both terms. In other words, it is possible to apply the interpretive rule here as well. Briefly stated, this interpretation could be phrased as: conversational pattern / participants in a conversation. I am here using "conversation" in a general and loose sense. Species interaction achieving a stable ecosystem can be thought of as the biological paradigm for a conversational domain. But human interactions can be similarly treated, as participants engaged in dialogue, whether with each other, with the environment, our with ourselves. This is the process underlying the conversational patterns that constitute the autonomous unity to which we belong and which we construct. We shall return to human cognition and conversational pattern in Chapters 15 and 16. I only wanted to point out that the star framework could be applied to the observer's properties as well, to see knowledge as an "it" generated through a process.

10.7 Holism and Reductionism

10.7.1

If we think of the philosophy of science, the duality holism/reductionism comes to mind as analogous to the material previously discussed in this chapter.

Most discussions place holism/reductionism in polar opposition (Smuts, 1925; Lazslo, 1972). This seems to stem from the historical split between empirical sciences, viewed as mainly reductionist or analytic, and the (European) schools of philosophy and social science that grope toward a dynamics of totalities (e.g., Kosik, 1969; Radnitsky, 1973). In the light of the previous discussion, both attitudes are possible for a given descriptive level, and in fact they are complementary. On the one hand, one can move down a level and study the properties of the components, disregarding their mutual interconnection as a system. On the other hand, one can disregard the detailed structure of the components, treating their behavior only as contributing to that of a larger unit. It seems that both these directions of analysis always coexist, either implicitly or explicitly, because these descriptive levels are mutually interdependent for the observer. We cannot conceive of components if there is no system from which they are abstracted; and there cannot be a whole unless there are constitutive elements.

10.7.2

It is interesting to consider whether one can have a measure for the degree of wholeness or autonomy of a system. One can, of course, always draw a distinction, make a mark, and get a "system," but the result does not always seem to be equally a "whole system," a "natural entity," or a "coherent object" or "concept." What is it that makes some systems more coherent, more autonomous, more whole, than others?

A first thing to notice is, that in the hierarchy of levels, "emergent" or "immanent" properties appear at some levels. For example, let us consider music as a system or organization of notes (for the purpose of this example, we do not attempt to reduce notes to any lower-level

distinctions). Then harmony only arises when we consider the simultaneous or parallel sounding of notes, and melody only arises when we consider the sequential sounding of notes. That is, harmony and melody are emergent properties of a level of organization above that of the notes themselves. Similarly, form can only emerge at a still higher level of organization, relating different melodic units to one another. These properties—form, melody, and harmony—are *systems properties*, arising from hierarchical organizations of notes into pieces of music; they are not properties of notes (Goguen, 1977). It also appears that "life" is an emergent property of the biological hierarchy of levels: it is nowhere to be found at the level of atoms and molecules, but it becomes clear at the level of cells through the autopoietic organization of molecules. Language can be seen as an emergent property at a still higher level of this hierarchy (Maturana, 1977). In general, organizational closure can be viewed as providing the mechanism through which emergent properties of new units arise, and thus as the "hinges" for the hierarchy of levels in natural systems. Thus, one point of view toward wholeness is that it co-occurs with interesting *emergent* properties at some level.

Another point of view toward wholeness, is that it can be measured by the *difficulty of reduction*: Because it is very hard to reduce the behavior of organisms to the behavior of molecules, we may say that organisms are whole systems. Similarly, it is very difficult (if not impossible) to reduce the effects of melodies to the effects of notes. One must consider properties of patterns of notes or molecules.

A third point of view is that a system is whole to the extent that its parts are *tightly interconnected*, that is, to the degree that it is difficult to find relatively independent subsystems. This is clearly related to the previous views. An interesting corollary of this view is that a system with a strongly hierarchical organization will be less whole than a system with a strongly heterarchical organization; that is, nets are more whole than trees. More precisely, given that the graph of connections of the parts of a system has no isolated subsystems, the more treelike it is, the less whole it is, while still being (presumably) a system. The extreme is probably a pure linear structure, without any branching at all.

A fourth point of view is that a system seems more whole if it is more *complex*, that is, more difficult to reduce to descriptions as interconnections of lower-level components. It is necessary in this discussion to take account of the (very modern) point of view that the more complex a system is to describe, the more *random* it is (Kolmogorov, 1968). Thus, for example, the wholeness of a living system is, in everyday encounters, construed as unpredictability. The more difficult it is to reduce a system to a simple input/output control, the more likely it is we will deem it alive. In this sense complete autonomy is logically equivalent to complete randomness. [Another example: a piece of music that is too complex, relative to our cultural expectations and inherent capacities, will sound random, chaotic, perhaps meaningless, but it will also sound whole. Here the extreme is white noise (Goguen, 1977).]

This viewpoint toward wholeness involves measurement relative to some standard interpreting system, an observer-community. But given such a standard, this viewpoint can be deduced from the preceding ones. For surely, it it is difficult to describe a system, it will also be difficult to reduce it to lower levels, and its parts will seem to be tightly interconnected. Quite possibly, its very complexity will appear as an emergent property. As Atlan has recently remarked in a fundamental paper, when randomness becomes "information" for a system depends strictly on the observer's position (Atlan, 1978). Different cognitive viewpoints might well be better able to process what now seems like a very complex system, and thus see it as less whole. Once again, the relativity to cognitive capacity appears.

10.7.3

These descriptive levels haven't been generally realized as complementary, largely because there is a difference between publicly announced methodology and actual practice in most fields of research in modern science. A reductionist attitude is strongly promoted, yet the analysis of a system cannot begin without acknowledging a degree of coherence in the system to be investigated; the analyst has to have an intuition that he is actually dealing with a coherent phenomenon. Although science has publicly taken a reductionist attitude, *in practice* both approaches have always been active. It is not that one has to have a holistic view as opposed to a reductionist view, or *vice versa*, but rather that the two views of systems are complementary.

Similar conclusions apply to the understanding of autopoiesis in relation to allopoiesis, or to symbolic descriptions as opposed to causal. Neither choosing one pole *against* the other nor treating them at the *same* level seems adequate; rather they must be acknowledged as distinct, but interdependent cognitive perspectives of the observer-community.

There is a strong current in contemporary culture advocating autonomy, information (symbolic descriptions), and holism as some sort of cure-all and as a radically "new" dimension. This is often seen in discussions about environmental phenomena, human health, and management. In this book we take a rather different view. We simply see autonomy and control, causal and symbolic explanations, reductionism and holism as complementary or "cognitively adjoint" for the understanding of those systems in which we are interested. They are intertwined in any satisfactory description; each entails some loss relative to our cognitive preferences, as well as some gain.

Sources

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