# An Allegory: The Solipsistic Self as the Hamiltonian of a Noninteracting Fermion

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Abstract. General intelligence functions in all relevant contexts. Similarly, absolute truth is true in all contexts. This suggests investigating general intelligence as inhabiting a metaphysics for absolute truth. We sketch a solipsistic metaphysics of human experience. Three levels of awareness, |- answering, |- questioning, |= investigating, are hypothesized to act as operators  $+1, +2, +3 \mod 8$  on eight mental contexts which are divisions of everything into k perspectives. We compare this with 2-fold complex and 8-fold real Bott periodicity in the representation of Clifford algebras, classification of super division algebras, embedding of Lie algebras, octonions, and tenfold way of topological insulators and superconductors. We identify a self with the Hamiltonian of a noninteracting free fermion. Quantum symmetries  $T \mid -, C \mid =$  and  $S \mid =$ define the Hamiltonian. We associate the division of everything into kperspectives with  $Cl_{0,k}$  and the k-th perspective with the linear complex structure  $J_k$ . We do not find evidence for operations  $+1, +2, +3 \mod 8$ . Instead, we find evidence for selfhood as a universal form, where the three levels of awareness perceive the self in terms of its |- accessibility,  $| \equiv$  inaccessibility and  $| \equiv$  definability.

Keywords: general intelligence · consciousness · Bott periodicity

## 1 Inhabiting Oneself

The quest for a metaphysical algebra precedes AGI research. Spencer-Brown, starting with a void, and then repeatedly applying a single operation (distinction), derived all of Boolean algebra. [21] In the 1990s, pioneers of consciousness studies discussed quaternions and octonions, coming up with psynet, a complex systems model of the mind.[5][6] They also contemplated Clifford algebras, which are classified by Bott periodicity, the subject of our paper.[20] We will interpret this mechanism as the conditions for selfhood, classified in terms of the quantum symmetries of the Hamiltonian of a noninteracting free fermion.

The psynet model identifies Bateson's learning triad (perception, thought, action) with the generators i, j, k of the quaternions.[3] Perception leads to thought leads to action leads to perception, just as ij = k, jk = i, ki = j. Octonions extend these to model all states of consciousness. The seven non-identity generators (which we write as 1, i, j, k, L, Li, Lj, Lk) are identified with the  $7 \pm 2$  entities

that can be held by working memory. The generator L is understood as an inner eye, whereby La denotes an inner observation of a mental process a, thus reflexive consciousness. This allows for learning about learning. This inner eye L has also been called a looking glass, in that it reverses factors, (La)b = L(ba).[9]

The psynet model predates the discovery, in 2009, of the tenfold way, the periodic table for topological insulators and superconductors, which presents Bott periodicity concretely in terms of three quantum symmetries: time reversal T, charge conjugation C and sublattice S.[10][19] C is also known as particle-hole symmetry and S as parity. These symmetries deem that a dynamical system would look the same if time was reversed, if matter and antimatter were swapped, or if the entire system was reflected in a mirror. They should inform the meta-physics of becoming (the arrow of time), being (what is and what is not) and learning (whether a learning cycle can be unlearned).

We explore the tenfold way, along with 2-fold complex and 8-fold real Bott periodicity, to consider whether it could model an 8-cycle of conceptual frameworks known as divisions of everything.[11][12] Bateson's triad is the division of everything into three perspectives and, in general, there are divisions into k perspectives,  $k = 0, 1, \ldots, 7$ . We further think of human experience as a dialogue between three minds, which is to say, three levels of awareness, for which these divisions serve as mental contexts.

In our analysis, quantum symmetries are the symmetries of context. What sets of circumstances, what regularities allow a dynamical system to have its own Hamiltonian, and thus a self-identity, a coherent time evolution? By analogy, in what ways do we assume or inhabit our very own selves?

We propose that a self is necessarily taken up in one of ten different contexts, making explicit one or more distinctions: defined S or not, accessible T or not, inaccessible C or not, unconsciously +1 or consciously -1, with regard to a specific me or an unspecified being or a universal void. We interpret the Hamiltonian as the self, quantum symmetries as levels of awareness, and 2-fold complex and 8-fold real Bott periodicity as the sequences by which a being comprehensively cycles through its understandings of its own self.

Our work-in-progress may be understood as an allegory. We first present the 3 minds and 8 mental contexts that we aspire to model, and then, as faithful pilgrims, we show where we think we see them in the mathematical garden of Bott periodicity and the tenfold way.

#### 2 Living as Three Minds

We suppose that human experience brings together three levels of awareness, which is to say, three minds: |-(yi)| answering, |-(ir)| questioning, |=(san)| investigating. (The characters are the numbers 1, 2, 3 in Chinese.) The first mind |-| unconsciously knows answers. The second mind |-| consciously does not know, thus asks questions. The third mind |=| balances the other two minds, so that the same information is in these two different forms, and then decides which of |-| and |=| should act in a given situation. We hypothesize these three

minds function in parallel, sharing a context  $|\bigcirc$  ("líng"), within which |- adds a perspective,  $|\_$  adds a perspective on a perspective, and  $|\equiv$  adds a perspective upon a perspective on a perspective.

Psychologists distinguish |- as fast-thinking, automatic, frequent, emotional, stereotypic, unconscious System 1 and |- as slow-thinking, effortful, infrequent, logical, calculating, conscious System 2. [8] We propose that |- is System 3, our full fledged consciousness, deliberately, willfully, cognizantly flying on two wings, |- and |-, balancing them, then choosing which one to lean on.

Introspectively, we experience |- as a savant, an oracle supplying us with images, sensations, associations, options, thoughts, inspirations, urges, moods, feelings, attitudes. We listen to |- as a prattle of words, and more generally, a conceptual language that offers form for carrying meaning. Both |- and |- have a life of their own, by which an individual performs scripted behavior and clichéd thinking. Additionally, like Plato's charioteer with their hand on the reins, |- establishes a definite context which juxtaposes |- and |-, setting course, adjusting ratios, adding emphasis, directing, punctuating, giving meaning.

Objectively, we observe the interplay of a material brain |-, the weighted averages of some 100 billion neurons which encode what a person knows, and a spiritual mind |- the propositions of a personal cognitive language with perhaps 100 thousand concepts, words, variables, slots that recode this in terms of what the person does not know. In the mind, a word such as *horse* functions as a question *What is a horse*? which receives an answer from the brain, a relevant memory of a horse. The history of AI likewise contrasts neural networks and symbolic processing. How might |= relate them?

The gap between |-, enmeshed in the world, and |-, distinct from it, can be analyzed in terms of action-perception loops. |- has a generative model of |-. As the environment changes, the success or failure of the model is communicated by |- to |- in terms of emotion, along with relevant sensations and candidate actions. Here |= functions as a brake to keep |- from updating its model prematurely. As explained by the theory of Active Inference, it is possible to update the environment rather than the model.[17] When |- works out its actions and |- responds with a state of peace, then |= releases its brake and |- reimposes its model as cognition. Emotion is the language from unconscious |- to conscious |-, cognition is the language in the opposite direction, and consciousness |= regulates the conversation.

As regards evolution, enactive |—draws from past experience, what it knows, predictive |—looks to the future, what it does not know, including dangers that one can never learn from but must steer clear of. Investigators of brain hemispheres have documented hundreds of examples of the tension between |—and |—in cultural history: yin-yang, sensation-intellect, romantic-rational, visual-verbal, intrinsic-extrinsic.[4][14]

Consciousness  $|\Xi|$ , as the cybernetic faculty for willfulness, deliberateness and cognizance, may be surprisingly simple to implement, so long as it coordinates a mind |- that knows and a mind |- that does not know. These levels of awareness act on a context. What are the possible contexts?

## 3 Cycling Through Eight Mental Contexts

Complex things are defined in terms of simple things. But how can we define the simplest things? They must define themselves in terms of their relationships with each other. This provides the mind with an abstract context, a complete set of perspectives to choose from.

We formalize a particular attempt to catalog such contexts called divisions of everything. [12] These can be understood as structures that carve up a void into a finite set of perspectives which are related by shifts in perspective as illustrated in Fig. 1. |-, |=, |= are hypothesized to be operators that lift the mind out of a context and into a metacontext, a new context with additional perspectives, 1, 2 or 3 modulo 8.

 $0^\infty$  nullsome is a context lacking perspectives, which is relevant for contemplating the void or simply God.

 $1^{\infty}$  onesome is a context with a single perspective, encompassing everything  $1_0$ , for contemplating order.



Fig. 1. Twosome, threesome, foursome, fivesome, sixsome, sevensome

 $2^{\infty}$  twosome is a context with 2 perspectives,  $2_1$  opposites coexist (as with free will) and  $2_0$  all is the same (as with fate), for contemplating existence. The mind shifts readily  $2_1 \rightarrow 2_0$  but not the other way around.

 $\mathbf{3}^{\infty}$  threesome is a context with 3 perspectives, a learning cycle of  $\mathbf{3}_0$  taking a stand,  $\mathbf{3}_1$  following through and  $\mathbf{3}_2$  reflecting, for contemplating participation. The cycle only turns forward  $\mathbf{3}_0 \rightarrow \mathbf{3}_1 \rightarrow \mathbf{3}_2 \rightarrow \mathbf{3}_0$ .

 $4^{\infty}$  foursome is for contemplating knowledge, which comes in four levels:  $4_0$  whether,  $4_1$  what,  $4_2$  how,  $4_3$  why. They accord with the context  $|\bigcirc$  and the three minds  $|\neg$ ,  $|\Box$ ,  $|\Xi$ . Knowing how  $|\Box$  shifts to knowing what  $|\neg$ ,  $4_2 \rightarrow 4_1$ , indicating that the brake on imposing cognition has been released, as discussed earlier, whereby knowing why  $|\Xi$  shifts to knowing whether  $|\bigcirc$ ,  $4_3 \rightarrow 4_0$ , letting go of consciousness.

 $5^{\infty}$  fivesome is for contemplating decision-making in terms of time or space. Looking backwards or looking outwards, every  $5_1$  effect has had its  $5_2$  cause of effect, but looking forwards or inwards, not every  $5_3$  cause has had its  $5_0$  effect of cause. And there is a critical point  $\mathbf{5}_4$  for deciding, the present or the boundary, with access to both directions.

 $\mathbf{6}^{\infty}$  sixsome is for contemplating morality. There are prods for relative learning:  $\mathbf{6}_0$  I follow through but do I reflect?  $\mathbf{6}_1$  I reflect but do I take a stand?  $\mathbf{6}_2$  I take a stand but do I follow through? And for absolute learning:  $\mathbf{6}_3$  Can I follow through absolutely?  $\mathbf{6}_4$  Can I reflect absolutely?  $\mathbf{6}_5$  Can I take a stand absolutely?

 $7^{\infty}$  sevensome is for contemplating logic, a self-standing consistent system which has slack, thus can be inhabited by a self-standing opposite such as what is true or good or known. This establishes a logical square with four corners and three sides, expressing a dialogue between what is known |-- and what is unknown |--.  $7_0$  All are known,  $7_1$  There exists a known,  $7_4$  All are known and there exists a known,  $7_3$  All are unknown,  $7_4$  There exists an unknown,  $7_5$  All are unknown and there exists an unknown,  $7_6$  There exists a known and there exists an unknown. The system may be empty, in which case the universal quantifier is satisfied but the existential quantifier is not.

 $\mathbf{8}^{\infty}$  eightsome would be a context with 8 perspectives, extending the sevensome with the missing side of the logical square: All are known and all are unknown. The system must be empty. All statements are true, as with a contradiction. The system collapses and  $\mathbf{8}^{\infty} \equiv \mathbf{0}^{\infty}$ .

## 4 Bott Periodicity of Clifford Algebras

In 1959, Bott proved the periodicity of the homotopy groups of the unitary group  $\pi_k(U) = \pi_{k+2}(U)$ , orthogonal group  $\pi_k(O) = \pi_{k+8}(O)$  and symplectic group  $\pi_k(Sp) = \pi_{k+8}(Sp)$ .



**Fig. 2.** Realizations of  $J_1, J_2, \ldots, J_8$ 

Concretely, we identify a perspective with a generator  $e_k$  of a real Clifford algebra, which in our case will typically be  $J_k$  with  $J_k^2 = -1$  and  $J_j J_k = -J_k J_j$ when  $j \neq k$ . We realize these as matrices in Fig.2 where each new  $J_k$  anticommutes with  $J_1, \ldots, J_{k-1}$ . The simplest real Clifford algebra is  $Cl_{0,0} \cong \mathbb{R}$ , one-dimensional. Adding a generator doubles the dimension, as with Pascal's

triangle, the calculus of choice, making  $Cl_{0,k} = Cl_{0,k-1} \oplus J_k Cl_{0,k-1}$  the sum of an even part and an odd part. Thus  $J_1 \equiv i$  yields the complex numbers  $Cl_{0,1} \cong \mathbb{C} = \mathbb{R} \oplus J_1\mathbb{R}$  and then  $J_2 \equiv j$  gives the quaternions  $Cl_{0,2} \cong \mathbb{H} =$  $\mathbb{C} \oplus J_2\mathbb{C} = \mathbb{R} \oplus J_1\mathbb{R} \oplus J_2\mathbb{R} \oplus J_1J_2\mathbb{R}$  where  $J_1J_2 \equiv ij = k$ . Adding generators and doubling further we get  $Cl_{0,3} \cong \mathbb{H} \oplus J_3\mathbb{H}$  and then matrix algebras  $Cl_{0,4} \cong M_2(\mathbb{H}), Cl_{0,5} \cong M_4(\mathbb{C}), Cl_{0,6} \cong M_8(\mathbb{R}), Cl_{0,7} \cong M_8(\mathbb{R}) \oplus M_8(\mathbb{R}), Cl_{0,8} \cong$  $M_{16}(\mathbb{R}).[7]$ 

We can also arrive here from  $Cl_{0,0} = \mathbb{R}$  by adding generators  $J_{+k}$  for which  $J_{+k}^2 = +1$ , yielding  $Cl_{1,0} \cong \mathbb{R} \oplus \mathbb{R}, Cl_{2,0} \cong M_2(\mathbb{R}), Cl_{3,0} \cong M_2(\mathbb{C}), Cl_{4,0} \cong M_2(\mathbb{H}), Cl_{0,5} \cong M_2(\mathbb{H}) \oplus M_2(\mathbb{H}), Cl_{0,6} \cong M_4(\mathbb{H}), Cl_{0,7} \cong M_8(\mathbb{C}), Cl_{0,8} \cong M_{16}(\mathbb{R}).$ These two sequences can be constructed using the recurrence relations  $Cl_{p+2,0} = M_2(\mathbb{R}) \otimes Cl_{0,p}$  and  $Cl_{0,p+2} = \mathbb{H} \otimes Cl_{p,0}.$ 

Typically, at this point, one establishes 8-fold periodicity by noting  $M_{16}(\mathbb{R})$ and  $\mathbb{R}$  are Morita equivalent in that the category of  $M_{16}(\mathbb{R})$ -modules is equivalent to the category of  $\mathbb{R}$ -modules. But then they are also equivalent to  $Cl_{2,0} \cong M_2(\mathbb{R})$ and  $Cl_{0,6} \cong M_8(\mathbb{R})$ . In understanding why structure collapses, we must consider how it was constructed.

The mechanics hinges subtly on the  $\mathbb{Z}_2$ -grading (even and odd degrees) of the Clifford algebras and their representations.  $Cl_{0,4} \cong Cl_{4,0} \cong \mathbb{H} \otimes M_2(\mathbb{R})$ , midway along the Bott clock, can be identified with  $\mathbb{H}$ , taken to be purely even. [23] Separately, we have the complex Clifford algebras  $Cl_0 = \mathbb{C}$  and  $Cl_1 = \mathbb{C} \oplus \mathbb{C}$ , where the scalars are complex numbers and we have 2-fold periodicity. This classifies the 3 division algebras  $Cl_{0,0} \cong \mathbb{R}, Cl_0 \cong \mathbb{C}, Cl_{0,4} \cong \mathbb{H}$  and 7 super division algebras  $Cl_{0,1} \cong \mathbb{R} \oplus J_1\mathbb{R}, Cl_{0,2} \cong \mathbb{C} \oplus J_2\mathbb{C}, Cl_{0,3} \cong \mathbb{H} \oplus J_3\mathbb{H}, Cl_{1,0} \cong \mathbb{R} \oplus J_{+1}\mathbb{R}, Cl_{2,0} \cong \mathbb{C} \oplus J_{+2}\mathbb{C}, Cl_{3,0} \cong \mathbb{H} \oplus J_{+3}\mathbb{H}, Cl_1 \cong \mathbb{C} \oplus iJ_1\mathbb{C}$ . Super division algebras express the same information in two forms, bosonic even A and fermionic odd JA, as with the two minds, |-| unmarked and |-] marked by the choice J. For any  $a \in A$  there is a unique  $a' \in A$  such that aJ = Ja'. Automorphism  $a \to a'$  fixes  $\mathbb{R}$  and like |-] matches |-] and |-].

The specific implementations illustrate the Chomsky hierarchy of automata. Linear  $rJ = Jr, r \in \mathbb{R}$ , antilinear  $cJ = J\overline{c}, c \in \mathbb{C}$ , and quaternionic  $hJ = xJx^{-1}h, x, h \in \mathbb{H}$  can be thought of as productions for finite automata  $J \to Jrr^{-1}$ , push down automata  $J \to cJc$  and linear bounded automata  $hJx \to Jhx$ . The latter implies  $hhhhJx \to Jxhhhh$ , the transport of computation. Turing machines can erase variables  $J \to \epsilon$ , making computations irreversible. We thus interpret  $Cl_{0,4} \cong \mathbb{H}$ , purely even, without J.

### 5 The Tenfold Way

We will try to model  $\mathbf{k}^{\infty}$  with the Clifford algebra  $Cl_{0,k}$ . A generator  $J_k$  is a choice that relates the three minds. Who experiences that choice?

The Hamiltonian is what quantum symmetries act upon. It indicates that a system has a coherent self-identity which can evolve in time. It aggregates the dynamical modes that a system can exhibit. The Hamiltonian equals its conjugate transpose, thus is Hermitian  $H = (H^*)^T = H^{\dagger}$ . As it acts on its context, so its context acts upon it.

We consider a fermionic system of L single particle states  $|i\rangle$  with creation operators  $\psi_i^{\dagger}$  and annihilation operators  $\psi_i$ . The Hamiltonian describes how multiple particles interact, assuming no two particles are in the same state. We focus on the interactions of a single particle with itself and its context.[1]

By Wigner's theorem, quantum symmetries are either linear or antilinear. They can also be usual (mapping particles to particles and holes to holes) or transposing (mapping particles to holes and holes to particles). We have four kinds: UL, UA, TL, TA. We assume UL symmetries are trivial and focus on the effects of context. There is at most one symmetry of each kind, up to equivalency.

A TA symmetry S acts on H with a unitary matrix  $U_SHU_S^{\dagger} = -H$ . A UA symmetry T and a TL symmetry C act on H with antiunitary matrices  $U_TK$ and  $U_CK$  where  $KHK^{-1} = H^*$ . We have  $U_THU_T^{\dagger} = H^*$  and  $U_CHU_C^{\dagger} = -H^*$ . For H, conjugation \* equals transposition T, swapping  $H_{ij}$  and  $H_{ji}$ , thus the processes  $|i\rangle \rightarrow |j\rangle$  and  $|i\rangle \leftarrow |j\rangle$ .  $H_{ij} = 0$  when  $|i\rangle, |j\rangle$  do not interact and  $H_{ij} \neq 0$  otherwise. For example, the states may form a ladder where  $H_{i,i+1} \neq 0$ . Eigenvector v and eigenvalue  $E_v$  isolate a dynamic mode  $Hv = E_v v$  whereby, for any state  $|j\rangle$ , the entries  $H_{ij}$  balance to keep the mode isolated. Transposing the matrix inverts these causal relationships, how the many balance the one. If creation operators  $\psi_i^{\dagger}$  form a column vector, and annihilation operators  $\psi_i$  form a row vector, then transposing H inverts causality, whether creation |-- causes annihilation |-- or vice versa. (In a single particle system, creation and annihilation must be balanced.) We interpret conjugation \* as alternating between stepping-in |-- and stepping-out  $|--\rightarrow -$  of a self or a perspective. But -H simply relabels the |-- positive energy particles as |-- negative energy holes and vice versa.

A quantum symmetry of a quantum symmetry,  $Q^2$ , is like the dual of a dual vector space, or the context of a context, which gives us the original being, but perhaps with a twist.  $Q^2 = +1$  models a 2-cycle, as with a being (organism) and its context (environment).  $Q^2 = -1$  extends the 2-cycle to a 4-cycle, making explicit 2 intermediate states, the perception from context to being, and the action from being to context, as with Active Inference. [17] Unconscious |-| lives the 2-state and conscious |-| the 4-state.

We juxtapose two expositions of the quantum symmetries in Fig.3 [22] gives a recipe for calculating the  $J_k$  as matrices, expresses T, C, S as products of them, and organizes this all with the 8-fold clock of Bott periodicity. [1] explicitly derives the Hamiltonians from T, C, S. We understand Hamiltonians in terms of local constraints given by  $J_k$  and global constraints imposed by T, C, S.

The matrices  $J_k$ , repeated along the diagonal as needed, are linear complex structures which realize a geometric form of the Bott clock, the Lie algebra embedding  $\mathfrak{o}(16r) \supset \mathfrak{u}(8r) \supset \mathfrak{sp}(4r) \supset \mathfrak{sp}(2r) \times \mathfrak{sp}(2r) \supset \mathfrak{sp}(2r) \supset \mathfrak{u}(2r) \supset \mathfrak{o}(2r) \supset \mathfrak{o}(r) \times \mathfrak{o}(r) \supset \mathfrak{o}(r)$ .  $J_k$  decomposes  $\mathfrak{g}_k = \mathfrak{h}_k + \mathfrak{m}_k$  in that we can express  $g \in \mathfrak{g}_k$  as g = h + m with  $h \in \mathfrak{h}_k, m \in \mathfrak{m}_k$ , where  $J_k h = hJ_k$  commute but  $J_k m = -mJ_k$  anticommute.  $\mathfrak{h}_k = \mathfrak{g}_{k+1}$  is a Lie algebra with which we continue

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the process. Multiplying m by the complex number i gives us our Hamiltonian. Setting  $\mathfrak{g}_0 = \mathfrak{o}(16r)$  and defining  $\mathfrak{g}_0 - \mathfrak{g}_k$  as everything, we get a division of everything into k perspectives,  $\mathfrak{g}_0 - \mathfrak{g}_k = \mathfrak{m}_0 + \mathfrak{m}_1 + \ldots \mathfrak{m}_{k-1}$ .

Fig.3 contrasts m and h at each stage, grouping matrix entries into  $2 \times 2$  blocks. Thus  $J_1$  distinguishes rotations a+bi and rotations times reflections. We interpret h as direct experience, what we know |-, and m as indirect reflection, what we don't know |-. The chain of embeddings delves ever deeper into our unconscious, retaining first hand knowledge h and discarding second hand knowledge m.



**Fig. 3.** Lie algebra decompositions  $\mathfrak{g}_k = \mathfrak{h}_k + \mathfrak{m}_k$ . We juxtapose our calculations based on [22] with the Hamiltonians in [1].

The  $16r \times 16r$  skew-symmetric matrices  $g \in \mathfrak{o}(16)$ , after 8 stages, get parsed away, so that each  $16 \times 16$  block is just a scalar  $aI, a \in \mathbb{R}$ . And  $\mathfrak{h}_8 \cong \mathfrak{o}(r) \cong \mathfrak{m}$ hints at the collapse  $\mathbf{8}^{\infty} = \mathbf{0}^{\infty}$  where all that is known equals all that is unknown,  $|-\equiv |-$ .

Can we recognize the structures  $\mathbf{k}^{\infty}$ ? Our  $J_k$  are products of  $J_1, J_2$ , the isometry  $L = J_3J_4$ , and possibly a diagonal matrix  $d_k$ . We get a pattern which resembles the octonions  $J_1, J_2, J_3 = d_3J_1J_2, J_4 = d_4LJ_1J_2, J_5 = d_5LJ_2, J_6 = d_6LJ_1, J_7 = d_7L$ . Here we first are adding  $J_1, J_2, J_1J_2$ , then we introduce a looking glass L to get  $J_4$ , and subsequently we are removing  $J_1$ , yielding  $J_5$ , or removing  $J_2$ , yielding  $J_6$ , or removing  $J_1J_2$ , yielding  $J_7$ . L brings to mind the structure of knowledge  $\mathbf{4}^{\infty}$ , which can be thought of as the framework shared by  $\mathbf{5}^{\infty}, \mathbf{6}^{\infty}, \mathbf{7}^{\infty}$ , and as a mirror after which the perspectives by which we don't know are supplemented with perspectives by which we know, until they are balanced. Note that the octonionic line bundle over  $\mathbb{OP}^1$  generates Bott periodicity.[2]

Can we recognize shifts in perspective? They may take different forms. The isometry L seems to function as the shift of shifts  $(\mathbf{4}_2 \rightarrow \mathbf{4}_1) \Rightarrow (\mathbf{4}_3 \rightarrow \mathbf{4}_0)$ .

The basic shift  $\mathbf{2}_1 \to \mathbf{2}_0$  may express the fact that we need a  $4 \times 4$  matrix to express  $J_2$  but only a  $2 \times 2$  matrix to express  $J_1$ . If we reduce the size, then we may not be able to increase it. A shift may also occur when we restrict the Hamiltonian to an eigenspace, which can happen once we have three generators, as a triple product squares to +1, as with  $(J_1J_2J_3)^2 = +1$ , which means that the operator  $J_1J_2J_3$  has eigenvalues +1 and/or -1. In the eigenspace  $V_{-1}$ , we have  $J_3 = J_1J_2, J_2 = J_3J_1, J_1 = J_2J_3$ , as desired for  $\mathbf{3}^{\infty}$ .



**Fig. 4.** Interpreting quantum symmetries  $T \models C \models$  and  $S \models$ 

Matching quantum symmetries T, C, S, Bott clocks and divisions of everything yields Fig.4. We may have no symmetries, or one symmetry, or all three symmetries, for by  $U_S = U_T U_C^*$ , any two imply the third. [15][16] Only two symmetries relate H with H directly, namely, the trivial lack of symmetries and also the self-standing symmetry S, which can be written  $U_S = \mathbf{1}_{p,q}$ , the diagonal matrix with p 1s and q -1s, so that  $\mathbf{1}_{p,q}H\mathbf{1}_{p,q} = -H$ . S establishes an absolutely defined frame, distinguishing what is fixed and what alternates, whereas the lack of symmetries signifies an undefined, frameless, selfless void.  $|\Xi|$  lives in generality, free of personality in this 2-fold complex Bott periodicity.

The other 8 symmetries relate H and  $H^*$ , thus live the self as me, relatively, able to step-in to live as |- or step-out to live as |-. When both T and C hold, then there is also S, which establishes the absolute frame  $\mathbf{1}_{p,q}$ , the natural basis for describing how the other two symmetries complement each other. We observe from [1][13] that the natural basis is given by S when it exists and otherwise by T and C when they stand independently. We take this to describe how  $S \mid \Xi$  brings together  $\mid-$  and  $\mid=$ .

For example, when  $T^2 = 1$ , we take  $U_T = \mathbf{1}_{p,q}$ ,  $\mathbf{1}_{p,q}H = H\mathbf{1}_{p,q}$ , and when  $C^2 = 1$ , we take  $U_C = \mathbf{1}_{p,q}$ ,  $\mathbf{1}_{p,q}H = -H\mathbf{1}_{p,q}$ . But when we take them together, then the natural reference point is  $U_S = \mathbf{1}_{p,q} = U_T U_C^*$ ,  $U_T = \mathbf{1}_{p,q}$ ,  $U_C = \mathbf{1}$ . When  $T^2 = -1, U_T = \mathbf{J}_m$ , and when  $C^2 = -1, U_C = \mathbf{J}_m$ , but when both hold, it is natural for us to take  $U_S = \mathbf{1}_{p,q} = U_T U_C^*$  where

$$U_T = \begin{pmatrix} \mathbf{J}_p \\ \mathbf{J}_q \end{pmatrix}, U_C = \begin{pmatrix} -\mathbf{J}_p \\ \mathbf{J}_q \end{pmatrix}, \mathbf{J}_p = \begin{pmatrix} 0 & \mathbf{1}_p \\ -\mathbf{1}_p & 0 \end{pmatrix}$$
(1)

Unconscious experience squares to +1 and conscious to -1. We examine  $\mathbf{k}^{\infty}$  to interpret T and C. C is unconscious regarding God  $\mathbf{0}^{\infty}$  and conscious regarding knowledge  $\mathbf{4}^{\infty}$ . T is unconscious regarding morality  $\mathbf{6}^{\infty}$  and conscious regarding existence  $\mathbf{2}^{\infty}$ . God is inaccessible and likewise knowledge concerns what is inaccessible, for we do not have direct access to Whether  $\mathbf{4}_0$  nor Why  $\mathbf{4}_3$ , and What  $\mathbf{4}_1$  and How  $\mathbf{4}_2$  are but intermediate constructs to get around that lack of access. Whereas our moral conscience is accessible, even if unconsciously, and matters of existence, what is possible and what is definite, are accessible consciously. Thus we identify T with |- accessibility, C with |- inaccessibility and S with |= definability. Selfhood manifests the combinations in Fig.4.

In terms of super division algebras, T relates |-- and |-- via a conscious choice  $J_2$  which has already been made, yielding two copies, as with  $\mathbb{C} \oplus J_2\mathbb{C}$ . Whereas C juxtaposes one copy with zero,  $\mathbb{R} \oplus 0$  and  $\mathbb{H} \oplus 0$ , indicating perhaps that the choice has yet to be made, and that the conscious mind dwells on the imbalance. Overall, the quantum symmetries suggest that the original hypothesis was mistaken in expecting that all 3 minds would act on all 8 mental states.

### 6 Underlying Selfhood

At this early, murky, allegorical stage of research, introspective alchemy could yet point the way to an AGI chemistry. We have identified three minds |answering, |= questioning,  $|\equiv$  investigating, with quantum symmetries T, C, S, and divisions of everything  $\mathbf{k}^{\infty}$  with Clifford algebras  $Cl_{0,k}$ . We have interpreted the tenfold way as the conditions for selfhood, and 8-fold real Bott periodicity as the template for consciousness, both relevant for AGI.

The brain has a body map and may also have a map of selfhood, carving up its global workspace into perspectives. Selfhood may emerge from ensembles of random matrices, or may be imposed or nurtured, as by love.

We considered a single particle interacting with itself and its context. This underlying selfhood may be a universal form. We can look for the Bott clock as an indicator of the three minds, for example, in octasulfur  $S_8$  (Z = 16), a possible catalyst for life, or in the Krebs cycle, a metabolic 8-cycle.

Acknowledgments. Warm thanks to ... for helpful conversations.

**Disclosure of Interests.** The authors have no competing interests to declare that are relevant to the content of this article.

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