M A T H 4 W I S D O M

$$\sum_{n=0}^{\infty} S_n(x)t^n = A(t)e^{xu(t)}$$

Sheffer Polynomials

Combinatorial Space for Quantum Physics

www.math4wisdom.com

Andrius Kulikauskas

PATREON

Schrödinger equation $$[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)]\psi = E_n\psi$$

Solution for Quantum harmonic oscillator involves Hermite polynomials

$$\psi_n = C_n P_n(kx)e^{-\frac{1}{4}(kx)^2}$$

Solution for Hydrogen atom (radial component) involves generalized Laguerre polynomials

$$\psi_{nl} = C_{nl} P_{nl}(kx)e^{-\frac{1}{2}kx}(kx)^l$$

Certain variants of these polynomials are Sheffer polynomials

$$S_n(x) = c_n P_n(rx)$$

Sheffer polynomials

$S_n(x)$ has degree n

Encodes a combinatorial space with n elements

Sheffer polynomials

- **1) Combinatorial space. Pointed partitions.**
- 2) The generating function
- 3) Calculating the k-th derivative
- 4) Bijection and induction
- 5) Significance

Bell numbers: Partitions of a set

$$1, 1, 2, 5, 15, 52, 203, 877, \ldots$$

5 ways to partition a set with 3 elements

$[012]$ $[0]\,[12]$ $[1]\,[02]$ $[2]\,[01]$ $[0]\,[1]\,[2]$

No choice where to put the initial element

$$[0]$$

We can ignore the initial element.

Rethink the initial part as distinguished,
as a base part in a pointed partition,
as free space...

Free space

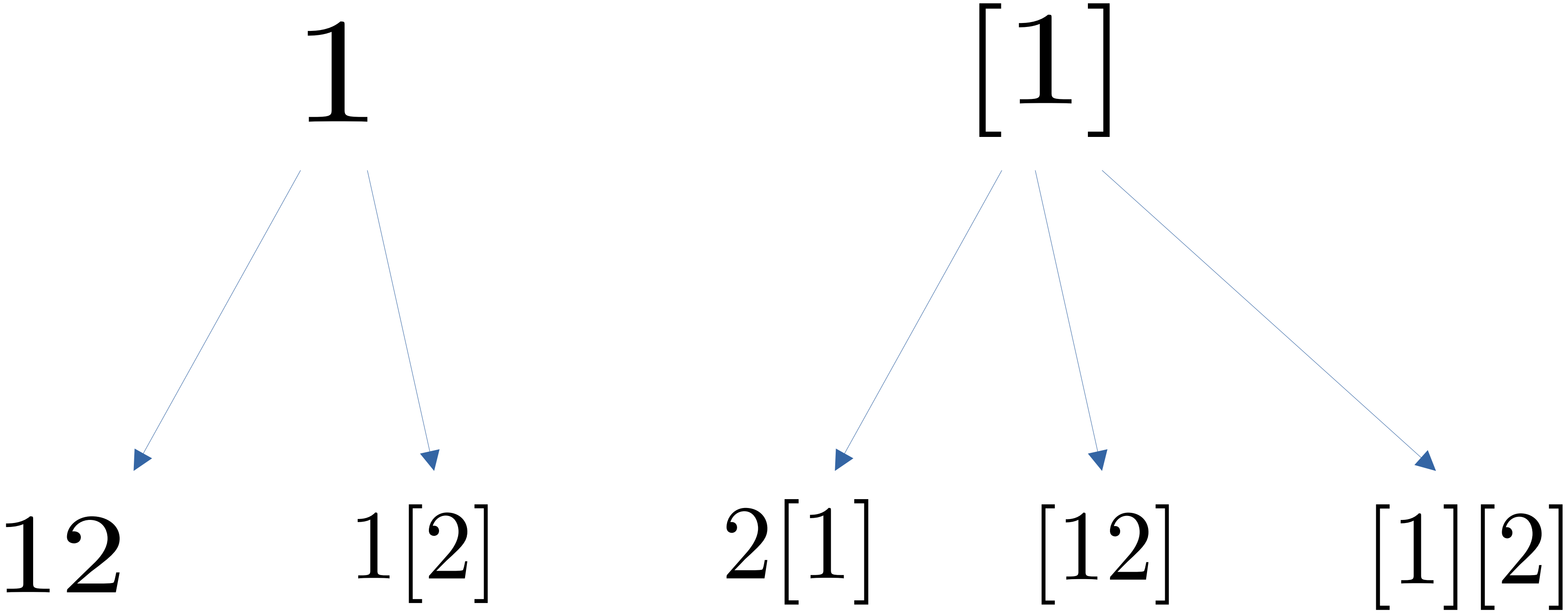
No compartment !

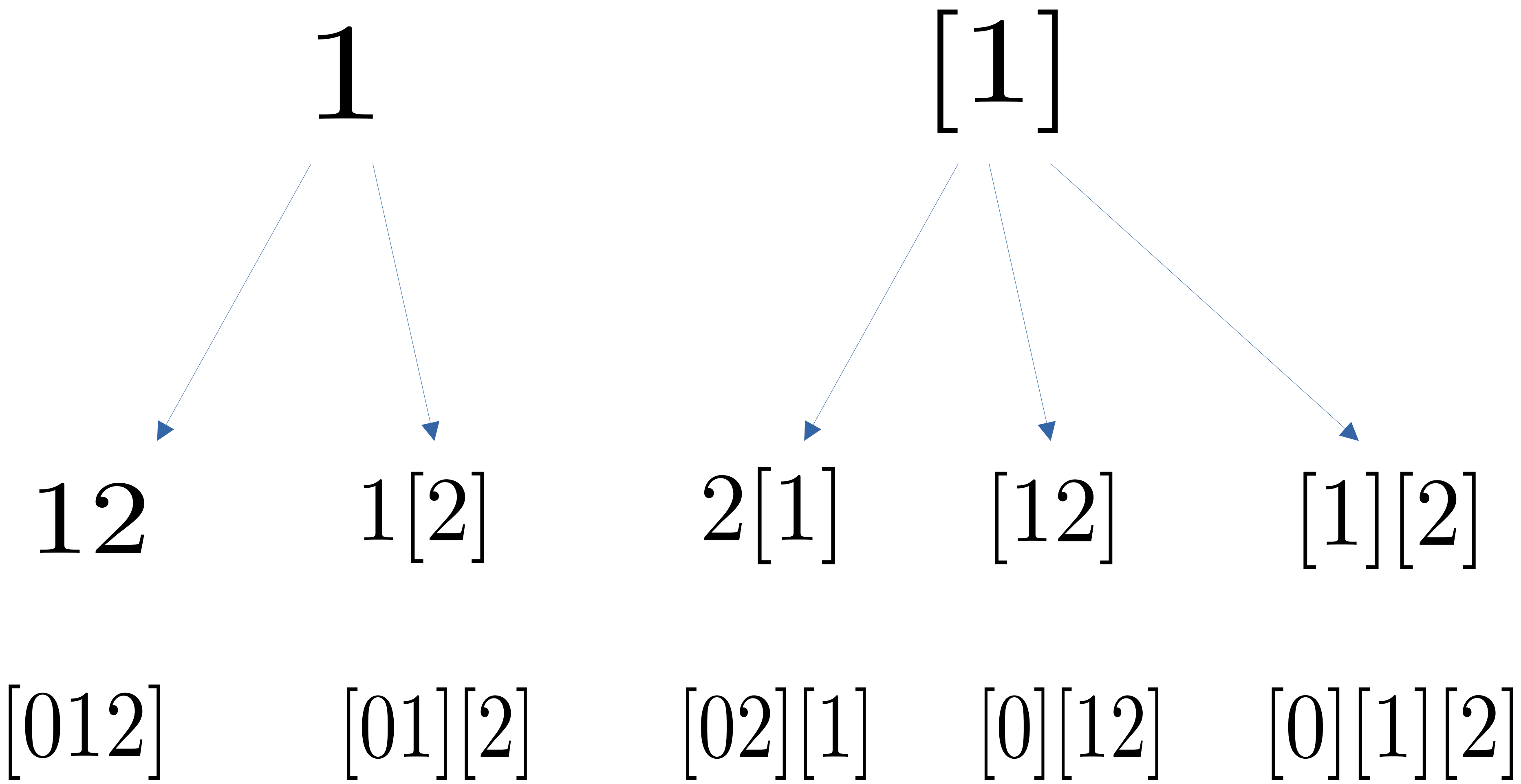
in free space

$$1$$

in a compartment

$$[1]$$


1
[1]
12
1[2]
2[1]
[12]
[1][2]


1
[1]
12
1[2]
2[1]
[12]
[1][2]
[012]
[01][2]
[02][1]
[0][12]
[0][1][2]

12	1[2]	2[1]	[12]	[1][2]

Add 3 to free space

123	13[2]	23[1]	3[12]	3[1][2]

Or add 3 to an existing compartment

	1[23]	2[13]	[123]	[13][2] [1][23]

Or add 3 to a new compartment

12[3]	1[2][3]	2[1][3]	[12][3]	[1][2][3]

3 ways to add the n+1st element

k compartments

n+1 elements

$$J_{k,n+1} = J_{k,n} + kJ_{k,n} + J_{k-1,n}$$

Add to free space

Or add to one of k existing compartments

Or add to a new compartment

Sheffer polynomials

- 1) Combinatorial space. Pointed partitions.
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Sheffer polynomials of type 0

$$\sum_{n=0}^{\infty} S_n(x)t^n = A(t)e^{xu(t)}$$

where

$$a_0 = 1 \qquad A(t) = \sum_{n=0}^{\infty} a_0 t^n = 1 + a_1 t + a_2 t^2 + \ldots$$

$$u_0 = 0, u_1 = 1 \qquad u(t) = \sum_{n=0}^{\infty} u_n t^n = t + u_2 t^2 + u_3 t^3 + \ldots$$

Sheffer $$\sum_{n=0}^{\infty} c_n P_n(x) t^n = \sum_{n=0}^{\infty} S_n(x) t^n = A(t) e^{x u(t)}$$

Hermite $$\sum_{n=0}^{\infty} \frac{1}{n!} \mathrm{He}_n(x) t^n = e^{-\frac{1}{2}t^2} e^{xt}$$
$$= (1 - t^2 + \frac{1}{2!}t^4 - \frac{1}{3!}t^6 + \dots) e^{xt}$$

Laguerre $$\sum_{n=0}^{\infty} L_n(-x) t^n = \frac{1}{1-t} e^{x \frac{t}{1-t}}$$
$$= (1 + t + t^2 + \dots) e^{x(t+t^2+t^3+\dots)}$$

Impressions...

global local

$$A(t) \quad e^{\,xu(t)}$$

free space compartments

$$n!S_n(x) = s_n x^n + \cdots + s_k x^k + \cdots + s_0$$

n compartments k compartments free space

$S_n(x)$ is the coefficent of t^n in

$$\sum_{n=0}^{\infty} S_n(x)t^n = A(t)e^{xu(t)}$$

$$= (1 + a_1 t + a_2 t^2 + \cdots)e^{x(t+u_2t^2+u_3t^3+\cdots)}$$

$$= (1 + a_1 t + a_2 t^2 + \cdots)(1 + xu(t) + \frac{1}{2!}(xu(t))^2 + \cdots)$$

$$= (1 + \cdots + a_{n-k}t^{n-k} + \cdots)(1 + \cdots + \frac{1}{m!}(xu(t))^m + \cdots)$$

contributes t^{n-k} contributes t^k

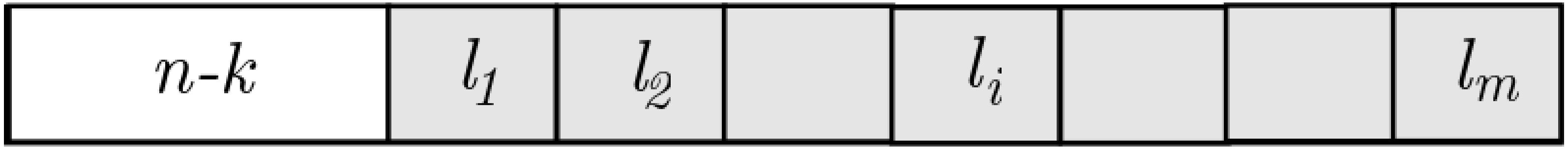
$$a_{n-k}t^{n-k}$$

$$\frac{1}{m!}(xu(t))^m$$

contributes t^{n-k}

contributes t^k

$$k = l_1 + l_2 + \cdots + l_m$$



free space

m compartments

$\frac{1}{m!}(xu(t))^m$ contributes $\frac{1}{m!}$ but what does it mean?

$\frac{1}{2!}$ unites products of 2 distinct parts such as

$t^2 \times t^4$ $t^4 \times t^2$

but doesn't work for repeated parts $t^3 \times t^3$

Rethink the generating function

$$\sum_{n=0}^{\infty} S_n(x)t^n = \sum_{n=0}^{\infty} \frac{1}{n!} P_n(x)t^n$$

Sheffer polynomials

- 1) Combinatorial space. Pointed partitions.
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- **3) Calculating the k-th derivative**
- 4) Bijection and induction
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$$u_0 = 0 \qquad u_1 = 1$$

$$\begin{aligned} u(t) &= u_0 + u_1 t + u_2 t^2 + \cdots \\ &= u(0) + u'(0)t + \frac{u''(0)}{2!}t^2 + \frac{u'''(0)}{3!}t^3 + \cdots \end{aligned}$$

$$u_i = \frac{u^{(i)}(0)}{i!}$$

multiply by $n!$

$$n! \times \frac{1}{m!}(xu(t))^m$$

$$n! \times \frac{x^m}{m!}\left(u'(0)t + \frac{u''(0)}{2!}t^2 + \frac{u'''(0)}{3!}t^3 + \cdots\right)^m$$

leads to terms with coefficients

$$\frac{n!}{(n-k)!l_1!l_2!\cdots l_m!} = \binom{n}{n-k, l_1, l_2, \ldots, l_m}$$

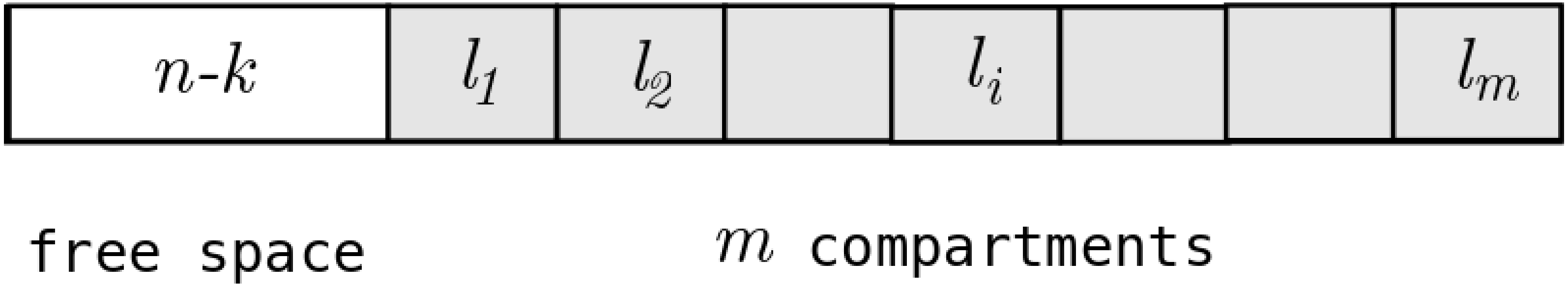
Factorial weights let us

distribute n elements across the $m+1$ compartments

$$n!\times \quad \frac{1}{(n-k)!} \quad \frac{1}{l_1!} \quad \frac{1}{l_2!} \quad \frac{1}{l_i!} \quad \frac{1}{l_m!}$$

free space m compartments

Elements make the compartments distinct.



$\frac{1}{m!}$ from $\frac{1}{m!}(xu(t))^m$ converts $m!$ lists to 1 set

Calculate $S_n(x)$ given $\sum_{n=0}^{\infty} S_n(x) t^n = A(t) e^{x u(t)}$

$$\frac{\mathrm{d}^n}{\mathrm{d}t^n} \sum_{k=0}^{\infty} S_k(x) t^k = \sum_{k=n}^{\infty} S_k(x) \frac{k!}{(k-n)!} t^{k-n}$$

$$\frac{\mathrm{d}^n}{\mathrm{d}t^n} \sum_{k=0}^{\infty} S_k(x) t^k \Bigg|_{t=0} = n! S_n(x)$$

$$\sum_{n=0}^{\infty} S_n(x) t^n = A(t) e^{x u(t)}$$

$$n! S_n(x) = \left. \frac{\mathrm{d}^n}{\mathrm{d}t^n} A(t) e^{x u(t)} \right|_{t=0}$$

$$S_n(x) = \frac{1}{n!} \left. \frac{\mathrm{d}^n}{\mathrm{d}t^n} A(t) e^{x u(t)} \right|_{t=0}$$

chain rule, product rule…

analytic symmetry

$$\frac{\mathrm{d}}{\mathrm{dt}} e^t = e^t$$

$$\frac{\mathrm{d}^4}{\mathrm{dt}^4} \sin t = \sin t$$

$$S_n(x) = \frac{1}{n!}\ \frac{\mathrm{d}^n}{\mathrm{dt}^n} A(t)e^{xu(t)}\bigg|_{t=0}$$

$$a_0 = A(0) = 1$$

$$u_0 = u(0) = 0 \qquad e^{xu(0)} = 1$$

$$u_1 = u'(0) = 1$$

$$S_0(x) = A(0)e^{xu(0)} = 1$$

$$E(t) = e^{xu(t)} \qquad\qquad E(0) = 1$$

$$E'(t) = xu'(t)e^{xu(t)}$$

mother function

$$\frac{\mathrm{d}}{\mathrm{dt}} E(t) = xu'(t)E(t)$$

baby function

$$\frac{\mathrm{d}}{\mathrm{dt}} A(t)E(t) = A'(t)E(t) + A(t)E'(t)$$

$$= A'(t)E(t) + A(t)xu'(t)E(t)$$

set $t = 0$

$$S_1(x) = 1!S_1(x) = A'(0)E(0) + A(0)xu'(0)E(0)$$

$$= A'(0) + x$$

$$\frac{d}{dt}A(t)B(t)E(t)$$

$$= A'(t)B(t)E(t) + A(t)B'(t)E(t) + A(t)B(t)E'(t)$$

2nd derivative yields

5 pedigrees

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} A(t)E(t)$$

$$= \frac{\mathrm{d}}{\mathrm{d}t}[A'(t)E(t) + A(t)xu'(t)E(t)]$$

$$= A''(t)E(t)$$

$$+A'(t)xu'(t)E(t)$$

$$+A'(t)xu'(t)E(t) + A(t)xu''(t)E(t) + A(t)(xu'(t))^2E(t)$$

5 pedigrees

$$A''(t)E(t) + A'(t)xu'(t)E(t) + A'(t)xu'(t)E(t) + A(t)xu''(t)E(t) + A(t)(xu'(t))^2E(t)$$

combine

$$A''(t)E(t) + 2A'(t)xu'(t)E(t) + A(t)xu''(t)E(t) + A(t)(xu'(t))^2E(t)$$

to yield 4 terms

$$A''(t)E(t) + 2A'(t)xu'(t)E(t) + A(t)xu''(t)E(t) + A(t)(xu'(t))^2E(t)$$

set $t = 0$

$$2!S_2(x) = A''(0) + (2A'(0) + u''(0))x + x^2$$

$$\frac{d^3}{dt^3}A(t)E(t)$$

$$= A'''(t)E(t) + 3A''(t)xu'(t)E(t) + 3A'(t)xu''(t)E(t) + 3A'(t)(xu'(t))^2E(t)$$

$$+A(t)xu'''(t)E(t) + 3A(t)xu'(t)xu''(t)E(t) + A(t)(xu'(t))^3E(t)$$

8 terms coming from 15 pedigrees

$$3!S_3(x) = A'''(0) + [3A''(0) + 3A'(0)u''(0) + A(0)u'''(0)]x$$

$$+[3A'(0) + 3A(0)u''(0)]x^2 + x^3$$

$$\frac{d^3}{dt^3}A(t)E(t)$$

$$= A'''(t)E(t) + 3A''(t)xu'(t)E(t) + 3A'(t)xu''(t)E(t) + 3A'(t)(xu'(t))^2E(t)$$

$$+A(t)xu'''(t)E(t) + A(t)(xu''(t))^2E(t) + 2A(t)xu'(t)xu''(t)E(t) + A(t)(xu'(t))^3E(t)$$

Each term has the form

$$N \cdot A^{(k)}(t)xu^{(l_1)}(t)xu^{(l_2)}(t)\cdots xu^{(l_m)}(t)E(t)$$

positive integer

father factor

baby factor

mother factor

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Sort: relevance | references | number | modified | created Format: long | short | data

A000110 Bell or exponential numbers: number of ways to partition a set of n labeled elements. +30 1204

(Formerly M1484 N0585)

1, **1, 2, 5, 15**, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, 1382958545, 10480142147, 82864869804, 682076806159, 5832742205057, 51724158235372, 474869816156751, 4506715738447323, 44152005855084346, 445958869294805289, 4638590332229999353, 49631246523618756274 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,3

COMMENTS

The leading diagonal of its difference table is the sequence shifted, see Bernstein and Sloane (1995). - N. J. A. Sloane, Jul 04 2015

Also the number of equivalence relations that can be defined on a set of n elements. - Federico Arboleda (federico.arboleda(AT)gmail.com), Mar 09 2005

a(n) = number of nonisomorphic colorings of a map consisting of a row of n+1 adjacent regions. Adjacent regions cannot have the same color. - David W. Wilson, Feb 22 2005

If an integer is squarefree and has n distinct prime factors then a(n) is the number of ways of writing it as a product of its divisors. - Amarnath Murthy, Apr 23 2001

Consider rooted trees of height at most 2. Letting each tree 'grow' into the next generation of n means we produce a new tree for every node which is either the root or at height 1, which gives the Bell numbers. - Jon Perry, Jul 23 2003

Sheffer polynomials

- 1) Combinatorial space. Pointed partitions.
- 2) The generating function
- 3) Calculating the k-th derivative
- **4) Bijection and induction**
- 5) Significance

bijection

pedigrees
of the terms of

$$\frac{\mathrm{d}^n}{\mathrm{dt}^n} A(t)E(t)$$

$\longleftrightarrow$

pointed
partitions of

$$\{1, \ldots, n\}$$

establish by **induction** on n

base case of induction $k = 0$

0-th derivative

$A(t)E(t)$

$\Longleftrightarrow$

empty free space

case $k=1$

1-st derivative
of $A(t)E(t)$

1 element
pointed partitions

$$A'(t)E(t)$$ ⟺ $$1$$
(in free space)

$$A(t)E'(t) =$$
$$A(t)xu'(t)E(t)$$ ⟺ $$[1]$$
(in compartment)

induction step from n to $n+1$

pedigrees of n-th derivative			**pointed partitions of** $\{1,\dots,n\}$
mother	$E(t)$		
father	$A^{(j)}(t)$	$\Leftrightarrow$	j elements in free space
baby	$xu^{(l_i)}(t)$	$\Leftrightarrow$	l_i elements in i-th compartment

$$\mathrm{A}^{(j)}(t)xu^{(l_1)}(t)xu^{(l_2)}(t)\cdots xu^{(l_m)}(t)E(t)$$
$$n = j + l_1 + l_2 + \cdots + l_m$$

Given n, extend to $n+1$

pedigrees of
n-th derivative

pointed partitions of
$\{1,\ldots,n\}$

$$n = j + l_1 + l_2 + \cdots + l_m$$

$\frac{\mathrm{d}}{\mathrm{dt}} A^{(j)}(t)$ $\Longleftrightarrow$ Add new element to free space

$\frac{\mathrm{d}}{\mathrm{dt}} xu^{(l_i)}(t)$ $\Longleftrightarrow$ Add new element to i-th compartment

$\frac{\mathrm{d}}{\mathrm{dt}} E(t)$ $\Longleftrightarrow$ Add new compartment with 1 element

Tracking distinct pedigrees

$$\begin{tikzcd}
& A(t)E(t) \longleftrightarrow \text{empty free space} \arrow[dl, "\#1"'] \arrow[dr, "\#1"] & \\
A^{\#1}(t)E(t) \longleftrightarrow 1 \arrow[d, "\#2"'] & & A(t)xu^{\#1}E(t) \longleftrightarrow [1] \arrow[d, "\#2"] \\
A^{\#1}(t)xu^{\#2}E(t) \longleftrightarrow 1[2] & & A^{\#2}(t)xu^{\#1}E(t) \longleftrightarrow 2[1]
\end{tikzcd}$$

Suppose bijection holds for n

pedigrees of
n-th derivative

pointed partitions of
$\{1, \dots, n\}$

$$n = j + l_1 + l_2 + \cdots + l_m$$

$\frac{\mathrm{d}}{\mathrm{dt}} A^{(j)}(t) \iff$ Add new element to free space

$\frac{\mathrm{d}}{\mathrm{dt}} x u^{(l_i)}(t) \iff$ Add new element to i-th compartment

$\frac{\mathrm{d}}{\mathrm{dt}} E(t) \iff$ Add new compartment with 1 element

Constructive proof needs lots of notation!

$$A^{\#1\#2\#3}(t)E(t) \leftrightarrow 123$$

$$A^{\#1}(t)xu^{\#2}xu^{\#3}E(t) \leftrightarrow 1[2][3]$$

$$A^{\#1\#2}(t)xu^{\#3}E(t) \leftrightarrow 12[3]$$

$$A^{\#2}(t)xu^{\#1}xu^{\#3}E(t) \leftrightarrow 2[1][3]$$

$$A^{\#1\#3}(t)xu^{\#2}E(t) \leftrightarrow 13[2]$$

$$A^{\#3}(t)xu^{\#1}xu^{\#2}E(t) \leftrightarrow 3[1][2]$$

$$A^{\#2\#3}(t)xu^{\#1}E(t) \leftrightarrow 23[1]$$

$$A(t)xu^{\#1\#2\#3}E(t) \leftrightarrow [123]$$

$$A^{\#1}(t)xu^{\#2\#3}E(t) \leftrightarrow 1[23]$$

$$A(t)xu^{\#1\#2}xu^{\#3}E(t) \leftrightarrow [12][3]$$

$$A^{\#2}(t)xu^{\#1\#3}E(t) \leftrightarrow 2[13]$$

$$A(t)xu^{\#1\#3}xu^{\#2}E(t) \leftrightarrow [13][2]$$

$$A^{\#3}(t)xu^{\#1\#2}E(t) \leftrightarrow 3[12]$$

$$A(t)xu^{\#1}xu^{\#2\#3}E(t) \leftrightarrow [1][23]$$

$$A(t)xu^{\#1}xu^{\#2}xu^{\#3}E(t) \leftrightarrow [1][2][3]$$

$$n!S_n(x) = \frac{\mathrm{d}^n}{\mathrm{d}t^n} A(t) e^{xu(t)} \Big|_{t=0}$$

$$A(0) = 1 \qquad u(0) = 0 \qquad u'(0) = 1 \qquad e^{xu(0)} = 1$$

$$n!S_n(x) = x^n + \cdots + s_k x^k + \cdots + A^{(n)}(0)$$

$[1][2]\ldots[n]$
n compartments

k compartments

$12\ldots n$
free space

Sheffer polynomials

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$$A^{(k)}(t)xu^{(l_1)}(t)xu^{(l_2)}(t)\cdots xu^{(l_m)}(t)E(t)$$

baby factor

father factor

mother factor

MANY

ALL

ONE

compare with minimization operator μ

THREESOME
(Learning cycle)

take a stand

follow through

reflect

ONE PARTICULAR
CONCEPTION

$$A(t) \longrightarrow e^{xu(t)}$$

ONE

$$xu'(t)$$

MANY

ALL

compare with Jacobi identity for Lie algebras

$$[X,[Y,Z]]+[Y,[Z,X]]+[Z,[X,Y]]=0$$

partition of $\{0, 1, 2, \ldots, n\}$ is better understood here as

pointed partition of $\{1, 2, \ldots, n\}$

$$n! S_n(x) = x^n + \cdots + s_k x^k + \cdots + A^{(n)}(0)$$

$[1][2] \ldots [n]$
n compartments
objective

k compartments
objective

$12 \ldots n$
free space
subjective

choosing 1 out of q

finite field with q elements

$$F_q$$

$$\binom{m}{r}_q = \frac{(1-q^m)(1-q^{m-1})\cdots(1-q^{m-r+1})}{(1-q)(1-q^2)\cdots(1-q^r)}$$

Gaussian binomial coefficient

counts subspaces

choosing 1 out of 1

mythical field with 1 element

$$F_1$$

$$\binom{m}{r} = \frac{m!}{r!(m-r!)}$$

binomial coefficient

counts subsets

$$u(t) = \sum_{i=0}^{\infty} u_i t^i$$

ordinary
generating function

$$u(t) = \sum_{i=0}^{\infty} \frac{u^{(i)}(0)}{i!} t^i$$

exponential
generating function

orthogonal Sheffer polynomials

$$S_n(x) = c_n P_n(x)$$

which values of c_n *are relevant?*

FIVESOME

orthogonal Sheffer polynomials

cause
Meixner

effect of cause
Meixner-Pollaczek

decision point
Laguerre

cause of effect
Charlier

effect
Hermite

Probability and statistics

NEF-QEV Natural Exponential Families
with Quadratic Variance Functions

Generalized Linear Models

See also:

Pearson distribution

PROBABILITY DISTRIBUTIONS

cause

Meixner

Negative binomial

effect of cause

Meixner-Pollaczek

Hyperbolic secant

decision point

Laguerre

Gamma

cause of effect

Charlier

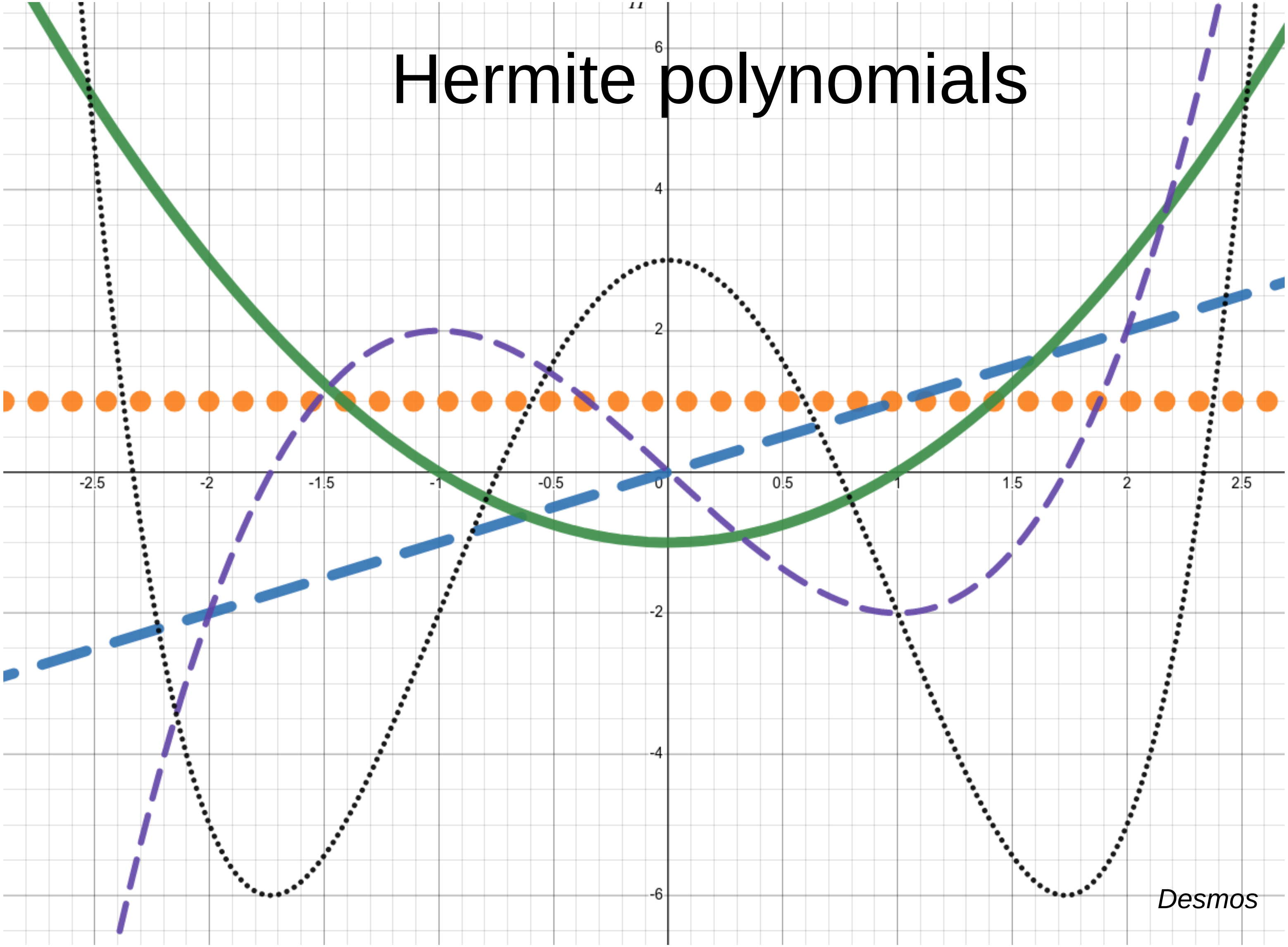
Poisson

effect

Hermite

Normal

Hermite polynomials



I have been studying the

Fundamental theorem of covering spaces

in Algebraic Topology with the

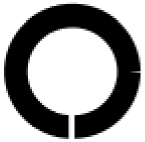
NYC Category Theory and Algebra Meetup

Join us!

universal covering space

pedigrees = distinct paths

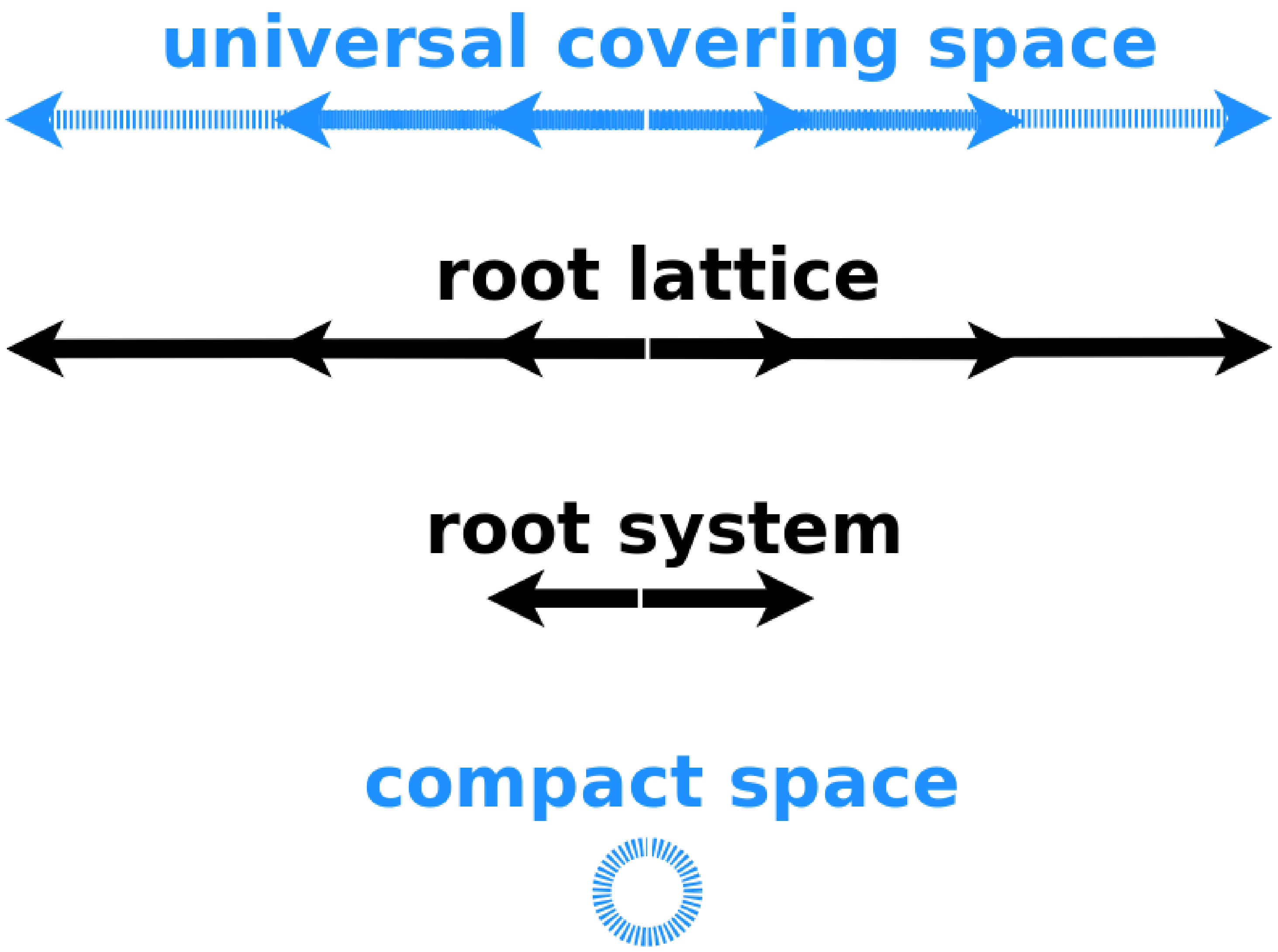
terms = loops = equivalences



space

YouTube:

John Baez’s lectures *This Week’s Finds*



universal covering space
root lattice
root system
compact space

A role for cellular automata?

Ask Stephen Wolfram

How to encode the evolution of physical laws?

Ask John Harland

Investigate math for wisdom?

Ask me! Andrius Kulikauskas **math4wisdom.com**

$$B_n = \sum_{k=0}^{n} \left\{ {n \atop k} \right\}$$

What is known about Sheffer polynomials and Stirling numbers of the second kind?

Ask Tian-Xiao He

2006 Tian-Xiao He *The Generalized Stirling Numbers, Sheffer-type Polynomials and Expansion Theorems*

FIVESOME

orthogonal Sheffer polynomials

cause
Meixner

effect of cause
Meixner-Pollaczek

decision point
Laguerre

cause of effect
Charlier

effect
Hermite

Thank you for helpful online resources!

1939 Sheffer *Some properties of polynomial sets of type zero* Duke Mathematical Journal archive.org

1973 Rota et al *Finite Operator Calculus* sciencedirect.com

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Thank you!

PATREON

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